

10.1

## 2-Proportion (Sample) Z Test

This test is used to compare 2 proportions from 2 independent samples

Ex Is the proportion of LN seniors attending college greater than the proportion of LC seniors

## Conditions (Both Samples)

- Random

- Normal Sampling Distributions

$$n_1 \hat{p}_1 \geq 10$$

$$n_2 \hat{p}_2 \geq 10$$

$$n_1 (1 - \hat{p}_1) \geq 10$$

$$n_2 (1 - \hat{p}_2) \geq 10$$

- Independent

$$N_1 > 10n_1$$

$$N_2 > 10n_2$$

# Hypothesis Test

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{\text{total \# success in both samples}}{\text{total \# of observations}}$$

pooled  
sample  
proportion

## Confidence Interval

$$CI = (\hat{p}_1 - \hat{p}_2) \pm Z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

↑  
Table B

## 2-PROPORTION Z-TEST

This test is used to compare proportions from 2 independent samples.

In a study done in Michigan, it was determined 38 (out of 62) poor children who attended pre-school needed social services later in life compared to 49 (out of 61) poor children who did not attend preschool.

Does this study provide significant evidence that preschool reduces the need for social services later in life?

P) IDENTIFY POPULATION PARAMETERS:  $(\hat{p}_1 = \frac{38}{62} = .6129, \hat{p}_2 = \frac{49}{61} = .8033)$

$p_1$  = proportion of preschooled children needing social services

$p_2$  = proportion of non-preschooled children needing social services

H) STATE HYPOTHESES:

$$H_0: p_1 = p_2 \quad (H_0: p_1 - p_2 = 0)$$

$$H_a: p_1 < p_2 \quad (H_a: p_1 - p_2 < 0)$$

A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) Random

Not known - results may not apply

b) Normal Sampling Distributions

$$n_1 \hat{p}_1 = (62) \left( \frac{38}{62} \right) = 38 > 10 \checkmark$$

$$n_2 \hat{p}_2 = (61) \left( \frac{49}{61} \right) = 49 > 10 \checkmark$$

$$n_1 (1 - \hat{p}_1) = (62) \left( \frac{24}{62} \right) = 24 > 10 \checkmark$$

$$n_2 (1 - \hat{p}_2) = (61) \left( \frac{12}{61} \right) = 12 > 10 \checkmark$$

c) Independent

$$N_1 > 10n_1 > 10(62) > 620 \text{ poor MI children in preschool?}$$

$$N_2 > 10n_2 > 10(61) > 610 \text{ poor MI children not in preschool?}$$

T) **PERFORM TEST USING**

a) **TABLE A:**

Calculate z-statistic and check Table:

$$\hat{p} = \frac{\text{total number of successes in both samples}}{\text{total number of observations in both samples}} = \frac{38 + 49}{62 + 61} = .7073$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.6129 - .8033}{\sqrt{(.7073)(.2927)\left(\frac{1}{62} + \frac{1}{61}\right)}} = -2.32$$

$$P\text{-value} = .0102 > \text{Table A}$$

$$\left[ \text{normalcdf}(-10, -2.32) \approx .0102 \right]$$

b) **CALCULATOR:**

STAT → TESTS → 2 Prop Z Test  $\begin{cases} Z = -2.32 \\ p = .0102 \end{cases}$

S) **STATE CONCLUSION:**

At  $\alpha = .05$ , we have evidence to reject  $H_0$  and conclude that preschooling poor children reduces the need for social services later in life

# Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



**CONFIDENCE INTERVAL:**

Construct a <sup>90%</sup>~~95%~~ confidence interval for the difference in proportions of people needing social services after attending preschool:

P) See above

A) See above

D) **Construct Interval**

a) **Using Formula**

$$CI = (\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$CI = (.6129 - .8033) \pm 1.645 \sqrt{\frac{(.6129)(.3871)}{62} + \frac{(.8033)(.1967)}{61}}$$
$$= (-.32, -.06)$$

b) **Using Calculator**

**STAT** → TESTS → 2 Prop Z Int → (-.32, -.06)

S) **State Conclusion** (Use *less* or *more*)

We are 90% confident that the percentage of poor children needing social services after attending preschool was between 6% and 32% less than those who did not attend preschool

Sec 10.2

## 2 Sample T Test

- Compares 2 means from 2 independent samples
- Use ANOVA to compare more than 2 means

Ex Is the average GPA of LN seniors greater than the average GPA of LC seniors?

## Conditions (Both Samples)

1) Random

2) Normal Sampling Distribution

a) Population normal

b) Large sample ( $n > 30$ )

c) Justification for normality ( $n < 30$ ) 

3) Independent

a) Samples themselves

b)  $N_1 > 10(n_1)$  and  $N_2 > 10(n_2)$

## Degrees of Freedom

Use smaller of  $(n_1 - 1)$  or  $(n_2 - 1)$

## Hypothesis Test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \rightarrow \text{Table B}$$

## Confidence Interval

$$CI = (\bar{X}_1, -\bar{X}_2) \pm t^* \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

↑ Table B

## Calculator Note

... Pooled: **No** yes

## 2-SAMPLE T TEST

*This test is used to compare 2 means from 2 separate (independent) samples.*

To compare the strength of Bounty paper towels to generic paper towels, 30 of each were randomly selected. Each paper was uniformly soaked with 4 ounces of water and while holding opposite edges of the towel, the number of quarters each paper towel could hold before ripping was counted. Here are the results:

<b>Bounty</b>	106	111	106	120	103	112	115	125	116	120	126
	125	116	117	114	118	126	120	115	116	121	113
	111	128	124	125	127	123	115	114			
<b>Generic</b>	77	103	89	79	88	86	100	90	81	84	84
	96	87	79	90	86	88	81	91	94	90	89
	85	83	89	84	90	100	94	87			

**Determine if Bounty paper towels are stronger than the generic brand at the  $\alpha = .01$  level.**

### P) STATE POPULATION PARAMETERS:

$M_1 = \text{avg \# quarters a wet Bounty paper towel can hold}$

$M_2 = \text{avg \# quarters a generic paper towel can hold}$

### H) STATE HYPOTHESES:

$H_0: M_1 = M_2$  ( $H_0: M_1 - M_2 = 0$ )

$H_a: M_1 > M_2$  ( $H_a: M_1 - M_2 > 0$ )

### A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) Random

Random samples taken

b) Normal sampling distribution

Since  $n_1$  and  $n_2$  are both large, Central Limit Theorem applies

c) Independent

1) Samples independent

2)  $N_B > 10(30) > 300$  sheets of Bounty ✓

$N_G > 10(30) > 300$  sheets of generic ✓

T) PERFORM TEST USING:

a) T Distribution Table:

i) Put data into lists and calculate x-bars/standard deviations (if necessary)

$$\bar{X}_1 = 117.6 \quad \bar{X}_2 = 88.13$$

$$S_{X_1} = 6.64 \quad S_{X_2} = 6.30$$

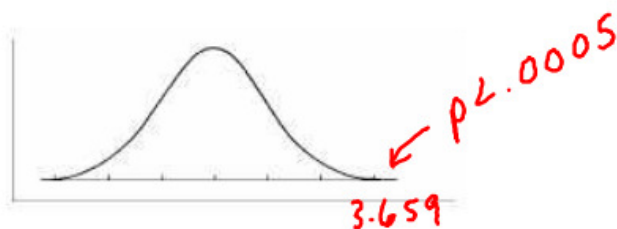
ii) Calculate t-statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{117.6 - 88.13}{\sqrt{\frac{(6.64)^2}{30} + \frac{(6.30)^2}{30}}} = 17.64$$

iii) Determine degrees of freedom:

$$n_1 = n_2 = 30 - 1 = 29$$

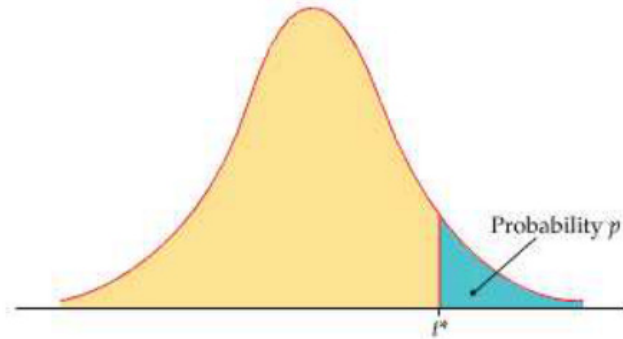
iv) Locate critical t-value and estimate P-value



$$t_{cdf}(17.64, 100, 29) = .00000000000000000002$$



Table entry for  $p$  and  $C$  is the critical value  $t^*$  with probability  $p$  lying to its right and probability  $C$  lying between  $-t^*$  and  $t^*$ .



**TABLE D**

**t distribution critical values**

df	Upper-tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.723
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$z^*$	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level $C$											

*critical t*

*t\**

2.462

3.659

b) CALCULATOR:

**STAT** → TESTS → 2-Samp T Test →  $t = 17.64$   
→  $2.9 \times 10^{-25}$

S) STATE CONCLUSION IN CONTEXT:

There is overwhelming evidence ( $p < .0005$ ) to reject  $H_0$  and conclude wet bounty paper towels hold more quarters than wet generic paper towels

CONFIDENCE INTERVAL:

Calculate a <sup>98%</sup>99% confidence interval for the mean difference in the number of quarters that a wet Bounty paper towel can hold compared to a wet generic paper towel.

P) See above

A) See above

D) Construct Interval:

a) Using Formula

$$CI = (\bar{x}_B - \bar{x}_G) \pm t^* \sqrt{\frac{(s_B)^2}{n_B} + \frac{(s_G)^2}{n_G}}$$

$$CI = (117.6 - 88.1) \pm 2.462 \sqrt{\frac{(6.64)^2}{30} + \frac{(6.30)^2}{30}}$$
$$= 29.5 \pm 4.11$$
$$= (25.39, 33.61) \star$$

$$1.4697 + 1.323$$
$$4.11434$$

b) Using Calculator

**TESTS** → 2-Samp T Int → (25.47, 33.46)

S) State Conclusion (Use less or more)

★ We are 98% confident that a wet bounty paper towel holds between 25 and 34 more quarters than a wet generic paper towel

Which Inference Test To Use?

