

Sec 11.1

# Inference Tests

Means / Averages →  $t$  Tests

Proportions →  $z$  Tests

Categorical Data →  $\chi^2$  Tests

↑ Chi-Square  
(Table c)

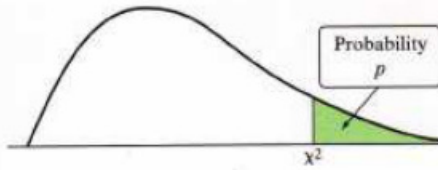


Table entry for  $p$  is the point  $\chi^2$  with probability  $p$  lying to its right.

Table C Chi-square distribution critical values												
df	$p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

## Types of Chi-Square Tests

- 1) Goodness of Fit (GOF)
  - 2) Homogeneity of Populations
  - 3) Independence / Association
- } Same

## Steps

~~P~~ No need to state parameter (s)

H Hypotheses in words

$H_0$ : The distributions are the same ...

$H_a$ : The distributions are different ...

## A Assumptions

- 1) Random sample
- 2) Every expected count  $\geq 5$
- 3) Independent  $(N > 10n)$

T Perform Test

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

} Uses degrees of freedom

S State conclusion in context

## CHI-SQUARE GOODNESS OF FIT TEST > GOF

This test is used to determine if observed counts are equal to a hypothesized distribution.

A researcher believes the Mars Company is misleading the public on its color distribution of M&Ms. He wants to compare the color distribution from a random sample of M&Ms to the Mars Company's expected values which are 10% brown, 20% red, 20% yellow, 10% green, 10% orange, 10% blue and 20% purple.

	Brown	Red	Yellow	Green	Orange	Blue	Purple
Sample	4	4	16	10	8	4	4
Expected	5	10	10	5	5	5	10

= 50

### H STATE NULL AND ALTERNATIVE HYPOTHESES:

$H_0$ : The color distributions of MMs are the same as the company claims

$H_a$ : The color distributions of MMs are different than the company claims

### A DETERMINE THAT CONDITIONS FOR TEST ARE ACCEPTABLE:

- ✓ Random - Says so
- ✓ Every expected count  $\geq 5$  See above  $\odot \odot$
- Independent  $N > 50(10) > 500$  MMs in world ✓

### T PERFORM TEST:

} GOF-Test on calculator?

$$\chi^2 = \frac{(4-5)^2}{5} + \frac{(4-10)^2}{10} + \frac{(16-10)^2}{10} + \frac{(10-5)^2}{5} + \frac{(8-5)^2}{5} + \frac{(4-5)^2}{5} + \frac{(4-10)^2}{10} = 18$$

b) Determine Degrees of Freedom = Number of Categories - 1 = 7 - 1 = 6

c) Determine P-Value

i) Using Table C:

$$P < .01$$

ii) Using calculator:

$$\boxed{\text{DISTR}} \rightarrow \chi^2 \text{cdf} (18, 100, 6) \rightarrow P = .006$$

### S STATE CONCLUSION IN CONTEXT

At  $\alpha = .01$ , there is very good evidence ( $P = .006$ ) to reject  $H_0$  and conclude the color distributions of MMs is different than what the company claims



## Note

To determine which color(s) are significantly different from expected, use 2-way table techniques (Chi) and/or z-tests for proportions

Sec 11.2

# Goodness of Fit (GOF)

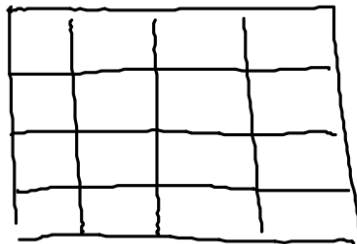


Homogeneity of Populations

2 (or more) samples

Association / Independence

1 Sample



## CHI-SQUARE TEST FOR HOMOGENEITY OF POPULATIONS

This test is used to determine if a single categorical variable has the same distribution in 2 (or more) distinct populations from 2 (or more) samples.

To determine if there was an association between race and opinions about schools, researchers surveyed 3 randomly selected groups of parents and asked them "Are high schools in your state doing an excellent, good, fair or poor job or don't you know enough to say?".

	Black Parents	Hispanic Parents	White Parents	TOTAL
Excellent	12	34	22	68
Good	69	55	81	205
Fair	75	61	60	196
Poor	24	24	24	72
Don't Know	22	28	14	64
TOTAL	202	202	201	605

### DETERMINE EXPECTED COUNTS:

$$\text{Expected Count} = (\text{Row Total})(\text{Column Total}) / \text{Sample Size}$$

	Black Parents		Hispanic Parents		White Parents	
	Actual	Expected	Actual	Expected	Actual	Expected
Excellent	12	22.7	34	22.7	22	22.6
Good	69	68.4	55	68.4	81	68.1
Fair	75	65.4	61	65.4	60	65.1
Poor	24	24.0	24	24.0	24	23.9
Don't Know	22	21.4	28	21.4	14	21.2

### H STATE NULL AND ALTERNATIVE HYPOTHESES

$H_0$ : There is no difference between race and opinions about schools

$H_a$ : There is a difference between race and opinions about schools

### A DETERMINE THAT CONDITIONS FOR TEST ARE ACCEPTABLE:

- ✓ Random - random samples used
- ✓ Every expected count  $\geq 5$  See Above ☺ ☺
- ✓ Independent  $N_B > 10(202) > 2020$ ,  $N_H > 10(202) > 2020$   
 $N_W > 10(201) > 2010$ ; Responses Independent ↓

## T PERFORM TEST USING...

### FORMULA/TABLE C:

a) Chi-Square Statistic:  $\chi^2 = \sum (O_i - E_i)^2 / E_i =$

$$\chi^2 = \frac{(12 - 22.7)^2}{22.7} + \frac{(69 - 68.5)^2}{68.5} + \dots + \frac{(14 - 21.3)^2}{21.3} = 22.43$$

b) Degrees of Freedom =  $(r - 1)(c - 1) = (5 - 1)(3 - 1) = 8$

Number of rows in table      Number of columns in table

c) P-Value

i) Table C

$P < .005$

} See Next Page

ii) Calculator:

$$\chi^2 \text{cdf}(22.43, 100, 8) = .004$$

### CALCULATOR:

a) Store observed counts in a [R,C] matrix: R C

**MATRIX** → EDIT → [A] → 5 × 3 → Enter Data

b) Perform  $\chi^2$  Test:

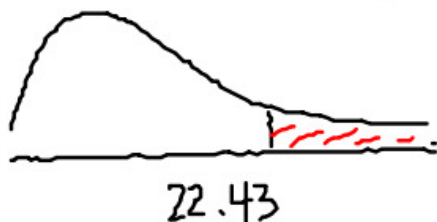
**STAT** → TESTS →  $\chi^2$  Test →  $\chi^2 = 22.4, P = .004$

### NOTE:

If [A] = Data then [B] = Expected Counts

## S STATE CONCLUSION IN CONTEXT:

There is significant statistical evidence ( $p = .004$ ) to reject  $H_0$  and conclude there is a difference between race and opinions about schools



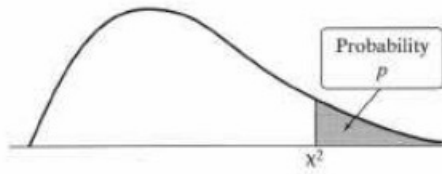


Table entry for  $p$  is the point  $\chi^2$  with probability  $p$  lying to its right.

Table C Chi-square distribution critical values												
df	P											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

## $\chi^2$ Test of Association / Independence

Is there a "significant" association between 2 categorical variables from the same sample?

## CHI SQUARE TEST OF ASSOCIATION/INDEPENDENCE

*This test is used to determine whether there is a significant association between 2 categorical variables from the **same sample**.*

To determine if there was a relationship between smoking status and socioeconomic levels, researchers categorized 356 federal male employees:

ACTUAL COUNTS	Socioeconomic Level		
	High	Middle	Low
Current Smoker	51	22	43
Former Smoker	92	21	28
Never Smoked	68	9	22

211

52

93

116  
 141  
 99  
 ---  
 356

### DETERMINE EXPECTED COUNTS:

Expected Count = (Row Total)(Column Total)/ Sample Size

EXPECTED COUNTS	Socioeconomic Level		
	High	Middle	Low
Current Smoker	68.75	16.94	30.30
Former Smoker	83.57	20.60	36.83
Never Smoked	58.68	14.46	25.86

### H STATE NULL AND ALTERNATIVE HYPOTHESES

H<sub>0</sub>: There is no association between SES and smoking in the population of federal male employees

H<sub>a</sub>: There is an association between SES and smoking in the population of federal male employees

### A DETERMINE THAT CONDITIONS FOR TEST ARE ACCEPTABLE:

? • Random - unknown; results may be invalid

✓ • Every expected count  $\geq 5$  See Above

? • Independent

N > 10 (356) > 3560 federal male employees



## T PERFORM TEST USING...

### FORMULA/TABLE C:

a) Chi-Square Statistic:  $\chi^2 = \sum (O_i - E_i)^2 / E_i$

$$\chi^2 = \frac{(51 - 68.75)^2}{68.75} + \dots + \frac{(22 - 25.86)^2}{25.86} = 18.51$$

b) Degrees of Freedom =  $(r - 1)(c - 1) = (3 - 1)(3 - 1) = 4$

Number of rows in table      Number of columns in table

c) P-Value

i) Table C:

$P < .001$

} See Next Page

ii) Calculator:

$$\chi^2 \text{cdf}(18.51, 100, 4) = .00098$$

### CALCULATOR:

a) Store observed counts in a [R,C] matrix:

**MATRIX** → EDIT → [A] → 3 × 3 → Enter counts

b) Perform  $\chi^2$  Test:

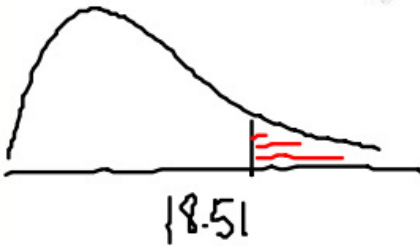
**TESTS** →  $\chi^2$  Test →  $\chi^2 = 18.51, p = .00098$

### NOTE:

If [A] = Observed Counts; [B] = Expected Counts

## S STATE CONCLUSION IN CONTEXT:

There is very strong evidence ( $p = .00098$ ) to reject  $H_0$  and conclude there is an association between smoking and socioeconomic status among federal male employees



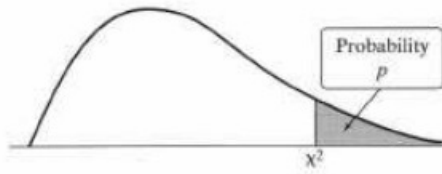


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3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2