Sec 11.1

ONE SAMPLE T-TEST - Gosset (students' t)

This test is used to compare a sample mean ($\bar{x}$) to a population mean (μ) or to determine a confidence interval for a population mean when ***σ is unknown****.*

Researchers believe that women (18-24) get less than the RDA of calcium (1200mg/day).

To test this hypothesis at the **α = .05 significance level**, an SRS of 38 women between the ages of 18 and 24 years estimated their daily intakes of calcium (in mg):

808	882	1062	970	909	802	374	416	784	997	651	716
438	1420	1425	948	671	696	1156	684	1933	748	1203	2433
1050	976	572	403	626	774	1253	549	1325	446	465	1269
1255	1100										

P) STATE POPULATION PARAMETER:

μ = avg daily intake of calcium for women 18-24 yo

H) STATE HYPOTHESES:

$H_0: \mu = 1200$ mg $H_a: \mu < 1200$ mg

A) VERIFY CONDITIONS REQUIRED FOR TEST:

✓ a) SRS - says so

$n > 40$

✓ b) Normal sampling distribution (normal population or large sample size or justification for normality after omitting outliers)

Sample size is large (CLT)

T) PUT DATA INTO LIST AND

a) USE TABLE C:

i) Determine mean ($\bar{x}$) and standard deviation (s) > 1-Var Stats

$\bar{X} = 926 \quad S = 427$

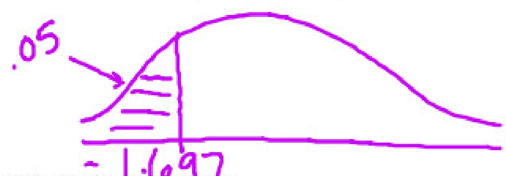
ii) Calculate t statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{926 - 1200}{\frac{427}{\sqrt{38}}} = -3.96$$

iii) Determine degrees of freedom

$df = n - 1 = 38 - 1 = 37$

iv) Determine critical t-value



Since $-3.96 < -1.697$

P-value $< .05$

b) USE CALCULATOR

$\boxed{\text{STAT}} \rightarrow$ TESTS $\rightarrow$ T-Test $\rightarrow t = -3.95$, $p = .0002$

$\boxed{\text{DISTR}} \rightarrow \text{tcdf}(-10, -3.96, 37) \rightarrow p = .0002$

min max df

S) STATE CONCLUSION:

There is significant evidence ($p < .05$) to reject H_0 and conclude that women (ages 18-24) do not get 1200 mg calcium / day

CONFIDENCE INTERVAL (Use PAIS): $CI = \bar{x} \pm t^* \frac{s}{\sqrt{n}}$

A 90% confidence interval for the mean daily intake in calcium can be found using:

STAT ---> TESTS ---> 8: T Interval = (809, 1043)

We are 90% confident that the average daily intake of calcium for women between the ages of 18 and 24 years old is between 809 mg and 1043 mg.

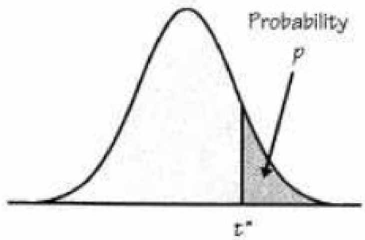


Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

TABLE C *t* **distribution critical values**

n-1

	Upper tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Experimental Designs

- Between Groups $\rightarrow$ Treatment
  $\rightarrow$ Control

- Within Groups / Matched Pairs

μ = avg difference (Before - After)

Context

$H_0: \mu = 0$

$H_a: \mu > 0$

$H_a: \mu < 0$

$H_a: \mu \neq 0$

MATCHED PAIRS **T TEST**

This test is used to compare the responses to a treatment in a ***within-groups*** *design (ie, does an SAT prep course improve an individual's SAT scores?).*

A listening test was administered to Spanish teachers before and after an institute designed to improve Spanish listening skills.

The maximum possible score on the test was 36:

Sub	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Pre	30	28	31	26	20	30	34	15	28	20	30	29	31	29	34	20	26	25	31	29
Post	29	30	32	30	16	25	31	18	33	25	32	28	34	32	32	27	28	29	32	32

Determine if the institute improved listening skills at the 5% significance level.

Use Lists?

CALCULATE THE DIFFERENCES BETWEEN THE 2 TREATMENTS:

Sub	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Pre	30	28	31	26	20	30	34	15	28	20	30	29	31	29	34	20	26	25	31	29
Post	29	30	32	30	16	25	31	18	33	25	32	28	34	32	32	27	28	29	32	32
Dif	-1	2	1	4	-4	-5	-3	3	5	5	2	-1	3	3	-2	7	2	4	1	3

P) STATE POPULATION PARAMETER:

μ = avg difference of listening scores for Spanish teachers attending institute (Post - Pre)

H) STATE HYPOTHESES:

$H_a: \mu = 0$ $H_a: \mu > 0$

A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) SRS - Unknown; results may be invalid

b) Normal sampling distribution- normal population or large sample size or justification for normal distribution after omitting outliers

- No outliers based on a modified box plot
- NPP appears linear → samp dist normal

T) PERFORM TEST:

a) USING TABLE C:

i) Determine mean ($\bar{x}$) and standard deviation (s)

$\bar{X} = 1.45 \quad S = 3.2032$

ii) Calculate t statistic

$$t = \frac{\bar{x} - 0}{\frac{s}{\sqrt{n}}} = \frac{1.45 - 0}{\frac{3.2032}{\sqrt{20}}} = 2.024$$

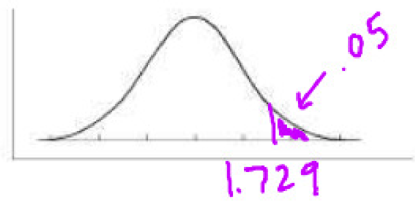
iii) Determine degrees of freedom

$df = 20 - 1 = 19$

iv) Determine critical t-value; P-value

Since $2.024 > 1.729$

the P-value $< .05$



b) USING CALCULATOR:

$\boxed{\text{STAT}}$ → TESTS → T Test → $t = 2.02$ → $p = .0286$

S) STATE CONCLUSION:

At $\alpha = .05$, there is evidence ($p = .0286$) to reject Ho and conclude the institute improves listening skills for Spanish teachers attending it

CONFIDENCE INTERVAL (Use PAIS):

A 90% confidence interval for the mean increase in listening scores can be found using:

STAT ---> TESTS ---> 8: T Interval = (.21, 2.69)

We are 90% confident that the mean increase in the listening scores was between .21 and 2.69 points after teachers participated in the institute.

Sec 11.2

2-Sample T Test

Compares 2 means from 2 independent samples

Conditions (for <u>each</u> sample)

- ✓ SRS
- ✓ Normal Sampling Distribution
- ✓ Independent Samples

Degrees of Freedom

- Use smaller of $n_1 - 1$ or $n_2 - 1$
- Actual formula on P. 659

Hypothesis Test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Confidence Interval

$$CI = (\bar{X}_1 - \bar{X}_2) \pm t^* \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

Misc

$\boxed{\text{STAT}}$ → TESTS ⋯→ Pooled → No / Yes

To compare more than 2 means → ANOVA

2-SAMPLE T TEST

This test is used to compare 2 means from 2 separate (independent) samples.

Below are an SRS of math SAT scores of 13-year olds
who took the test between 1980 and 1982:

Group	n	x-bar	s
Males	883	416	87
Females	937	386	74

Determine if male scores are significantly higher than female scores at the $\alpha = .01$ level.

P) STATE POPULATION PARAMETERS:

$\mu_1 = \mu_M$ = Avg Math SAT score of 13 yo males (1980-82)

$\mu_2 = \mu_F$ = Avg Math SAT score of 13 yo females (1980-82)

H) STATE HYPOTHESES:

$H_0: \mu_M = \mu_F$ $H_a: \mu_M > \mu_F$

$(H_0: \mu_M - \mu_F = 0)$ $(H_a: \mu_M - \mu_F > 0)$

A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) ✓ SRS?

Both samples come from an SRS

b) ✓ Normal sampling distribution?

Both sample sizes are large (CLT)

T) PERFORM TEST USING

a) TABLE C:

i) Put data into lists and calculate x-bar/standard deviation (if necessary)

2-Var Stats ?

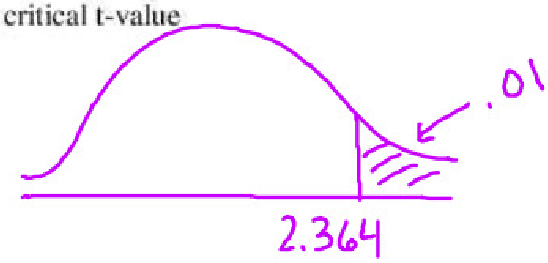
ii) Calculate t-statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{416 - 386}{\sqrt{\frac{87^2}{883} + \frac{74^2}{937}}} = 7.90$$

iii) Determine degrees of freedom:

$n - 1 = 883 - 1 = 882$

iv) Locate critical t-value



b) CALCULATOR:

$\boxed{\text{STAT}} \rightarrow$ TESTS $\rightarrow$ 2 SampTTest $\rightarrow$ $p < .01$

S) STATE CONCLUSION IN CONTEXT:

There is overwhelming evidence ($p < .0001$) to reject Ho and conclude that 13 yo males did better on the Math SAT than 13 yo females between 1980 and 1982

CONFIDENCE INTERVAL (Use PAIS):

A 98% confidence level for the mean difference in SAT math scores between males and females can be found using:

STAT ---> TESTS ---> 0: 2-Samp T Int = (21.16, 38.84)) 1 Question ≈ 10 SAT

We are 98% confident that 13-year old males scored between 21 and 39 points higher on the SAT math test than 13-year old females between 1980 and 1982.