Chapter 5

Section 5.1

Check Your Understanding, page 286:

1. (a) This probably means that if you asked a large sample of U.S. adults whether they usually eat breakfast, about 61% of them will answer yes. (b) In a random sample of 100 adults, we would expect that around 61 of them will usually eat breakfast. However, the exact number will vary from sample to sample.
2. (a) This probability is 0. If an outcome can never occur then it will occur in 0% of cases. (b) This probability is 1. If an outcome will occur on every trial, then it will occur in 100% of cases. (c) This probability is 0.01. If an outcome is very rare, but will occur once in a while in a long sequence of trials, this is consistent with a probability of 0.01 which means that the event occurs in about 1% of trials. (d) This probability is 0.6. An outcome that will occur more often than not will occur in more than 50% of trials which means a probability that is greater than 0.50. This leaves us with two choices: 0.60 and 0.99. The wording suggests that the event occurs often but not nearly every time. This suggests something that occurs 60% of the time.

Check Your Understanding, page 292:

1. Assign the members of the AP Statistics class the numbers 01-28 and the rest of the students numbers 29-95. Again ignore the numbers 96-99 and 00. In Table D read off 4 two-digit numbers, ignoring duplicates. Record whether all four numbers are between 01-28 or not. Do this many times and compute the percent of time that that all four students are part of the AP Statistics class.
2. Assign the numbers 1-10 to Jeff Gordon, 11-40 to Dale Earnhardt Jr., 41-60 to Tony Stewart, 61-85 to Danica Patrick, and 86-99 and 00 to Jimmie Johnson. Do RandInt(1,100) until you have at least one number (box) for each of the 5 drivers. Count how many numbers (boxes) you had to sample in order to get at least one of each.

Exercises, page 293:

5.1 (a) If we use a polygraph machine on many, many people who are all telling the truth, about 8% of the time, the machine will say that the people are lying. (b) Answers will vary. A false positive would mean that a person telling the truth would be found to be lying. A false negative would mean that a person lying would be found to be telling the truth. The U.S. judicial system is set up to think that a false positive would be worse – that is, saying that someone is guilty (lying) who is not is worse than finding someone to be truthful when they are, in fact guilty (lying).

5.2 (a) If we test many, many athletes who have not taken performance-enhancing drugs, about 3% of the time the test will say that they have. (b) Answers will vary. One answer would be that the false positive is worse because we will ruin the career of a good athlete who has, in fact, been following the rules.

5.3 (a) If we look at many families where the husband and wife both carry this gene, in approximately 25% of them the first-born child will develop cystic fibrosis. (b) If the family has 4 children, this constitutes a sample of size 4 which is very small. In order for the probability to be closely reflected in the sample, the sample size must be very large.

Short Run vs Long Run

5.4 (a) If we look at many, many hands of poker in which you hold a pair, the fraction of times in which you can make four of a kind will be about 88/1000. (b) It does not mean that exactly 88 out of 1000 such hands would yield four of a kind; that would mean , for example, that if you've been dealt 999 such hands and only had four of a kind 87 times, then you could count on getting four of a kind the next time you held a pair.

5.5 (a) Answers will vary. For example, on one set of 25 spins, we obtained 16 tails and 9 heads. Based on this set of 25 spins the probability of heads is $\frac{9}{25} = 0.36$. (b) You could get an even better estimate by spinning the coin many more times.

5.6 (a) Answers will vary. For example, on one set of 25 observations, we obtained 11 tails and 14 heads. Based on this set of 25 observations the probability of heads is $\frac{14}{25} = 0.56$. (b) You could get an even better estimate by knocking down the coin many more times.

5.7 In the short run there was quite a bit of variability to the percentage of free throws made. In fact, the basketball player did not do as well early on, but his percentage both increased somewhat and became less variable as time went on.

5.8 In the short term there was lots of variability to the proportion of heads. In the long term this proportion settles down around 0.50.

5.9 No, the TV commentator is incorrectly applying the law of large numbers to a small number of at bats for the player.

5.10 No, the weather in various years may not be independent. Plus the TV weather man is applying the law of large numbers to a small number of years which is an incorrect application of it.

5.11 (a) There are 10,000 four-digit numbers (0000, 0001, 0002,…, 9999), and each is equally likely to be chosen. One way to see this is to consider writing each of the numbers on a slip of paper and putting all of the numbers in a big box. Then you randomly select any slip of paper. Each of the 10,000 slips in the box, including the ones with 2873 and 9999 on them, is equally likely to be the one you select. (b) Most people would say that 2873 is more likely than 9999 to be randomly chosen. To many it somehow "looks" more random – we don't "expect" to get the same number four times in a row. Choose a number that most would avoid so that if you win, you don't have to split the pot with as many other people.

5.12 (a) The wheel is not affected by its past outcomes—it has no memory; outcomes are independent. So on any one spin, black and red remain equally likely. (b) The gambler is wrong again. Removing a card changes the composition of the remaining deck, so successive draws are not independent. If you hold 5 red cards, the deck now contains 5 fewer red cards, so your chance of another red decreases.

5.13 (a) Let 1, 2, and 3 represent the player making the free throw and 4 represent a miss. If 5 or 6 comes up, ignore it and roll again. (b) Let the two-digit numbers 01-75 represent making the free throw and 76-99 and 00 represent missing. Read a two-digit number from Table D. (c) Let diamonds, spades, and clubs represent making a free throw and hearts represent missing. Deal one cared from the deck.

5.14 (a) Let 1 and 2 represent a green light and 3-6 represent a red light. Roll the die once. (b) Let the numbers 1, 2 and 3 represent a green light and 4-9 represent a red light. Ignore 0. Look up one number in the table. (c) Let diamonds represent a green light and clubs and spades represent a red light. Deal one card. If a heart is dealt, ignore that outcome and deal again.

5.15 (a) There are actually 19 numbers between 00 and 18, 19 numbers between 19 and 37, and 3 numbers between 38 and 40. This changes the proportions between the three different outcomes. (b) There is no reason to skip numbers that have already been encountered in the table. These numbers just represent the handedness, not a particular individual to select for the sample.

5.16 (a) There is no reason to skip numbers that have already been encountered in the table. These numbers just represent the obesity, not a particular individual to select for the sample. (b) This will give the numbers 0 through 9 an equal chance of occurring, but if boys and girls are equally likely, we would expect it to be more likely to have 4, 5 or 6 boys than 0 or 9.

5.17 (a) This is a legitimate simulation. The chance of rolling a 1, 2 or 3 is 75% on a 4-sided die and the rolls are independent of each other. (b) This is not a valid design because the chance of heads is 50% (assuming the coin is fair) rather than the 60% that she hits the center of the target. This will underestimate her percent of hitting the target.

5.18 (a) This is not a valid design because you are not putting the card back in the deck after dealing. This means that on the second draw, the proportion of red and black cards has changed, depending on what card was dealt first. (b) This is an appropriate design because there is a 95% chance of getting a number between 00 and 94 and the selections are independent because Table D was used.

5.19 (a) What is the probability that, in a random selection of 10 passengers, none from first class are chosen? (b) Number the first class passengers as 01-12 and the other passengers as 13-76. Ignore all other numbers. Look up two-digit numbers in Table D until you have 10 unique numbers (no repetitions because you do not want to select the same person twice). Count the number of two-digit numbers between 01 and 12. (b) The numbers read in pairs are: **71 48 70** 99 84 **29 07** 71 48 **63 61 68 34** 70 **52**. The bold numbers indicate people who have been selected. The other numbers are either too large (over 76) or have already been selected. There is one person among the 10 selected who is in first class in this sample. (d) Since in 15% of the samples no first class passenger was chosen, it seems plausible that the actual selection was random.

5.20 (a) What is the probability that, in selecting 7 tiles from 100, all 7 are vowels? (b) Let the numbers 01-42 represent the vowels, 43-98 represent the consonants, and 99 and 00 represent the blank tiles. Look up two-digit numbers in Table D until you have 7 unique numbers (no repetitions since once you pull one tile from the bag you cannot pull it again). Record whether all 7 numbers are between 01 and 42 or not. (c) The numbers read in pairs are: **00 69 40 59 77 19 66**. The bold numbers indicate tiles that have been selected. In this case only 2 are vowels, so the whole sample is not just vowels. (d) Since there were only 2 occasions among 1000 repetitions in which all tiles were vowels, that suggests that this would only happen about 0.2% of the time. It is likely that the bag was not mixed properly.

5.21 (a) Read off 30 three-digit numbers from the table, ignoring numbers greater than 365 and 000. Repeats are allowed because these numbers represent the actual birth date of an individual. Record whether there were any repeats in the sample or not. (b) Answers will vary. One possible answer: We used Minitab to select 5 samples. There were repeats in all 5 samples. (c) Answers will vary. One possible answer: After the simulation we would not be surprised that the probability is 0.71 since we found repeats in 100% of our samples. Before the simulation this may seem surprising since there are 365 different days that people could be born on.

5.22 (a) Answers will vary. One possible answer: In our sample we won 11 out of 25 times when we stayed and we won 14 out of 25 times when we switched. (b) Answers will vary. Based on our simulation we might suggest that the readers were correct. We won 56% of the time when we switched

→ RandInt(1, 365, 30) → $\boxed{\text{STO}}$ → L_1 → Sort

$200	$100	$50
01 - 05	06 - 20	21 - 50 ✓
01 - 10	11 - 40	41 - 00
1	2 - 4	5 - 0 (1:3:6)

3. Every Monday a local radio station gives coupons away to 50 people who correctly answer a question about a news fact from the previous day's newspaper. The coupons given away are numbered from 1 to 50, with the first person receiving coupon 1, the second person receiving coupon 2, and so on, until all 50 coupons are given away. On the following Saturday, the radio station randomly draws numbers from 1 to 50 and awards cash prizes to the holders of the coupons with these numbers. Numbers continue to be drawn without replacement until the total amount awarded first equals or exceeds $300. If selected, coupons 1 through 5 each have a cash value of $200, coupons 6 through 20 each have a cash value of $100, and coupons 21 through 50 each have a cash value of $50.

   (a) Explain how you would conduct a simulation using the random number table provided below to estimate the distribution of the number of prize winners each week.

   (b) Perform your simulation 3 times. (That is, run 3 trials of your simulation.) Start at the leftmost digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table. Report the number of winners in each of your 3 trials.

$50 $50 $200

3	72749	13347	65030	26128	49067	02904	49953	74674	94617	13317	) 3 - 3 - 3
	81638	36566	42709	33717	59943	12027	46547	61303	46699	76423	→ 5
	38449	46438	91579	01907	72146	05764	22400	94490	49833	09258	7 4

GO ON TO THE NEXT PAGE.

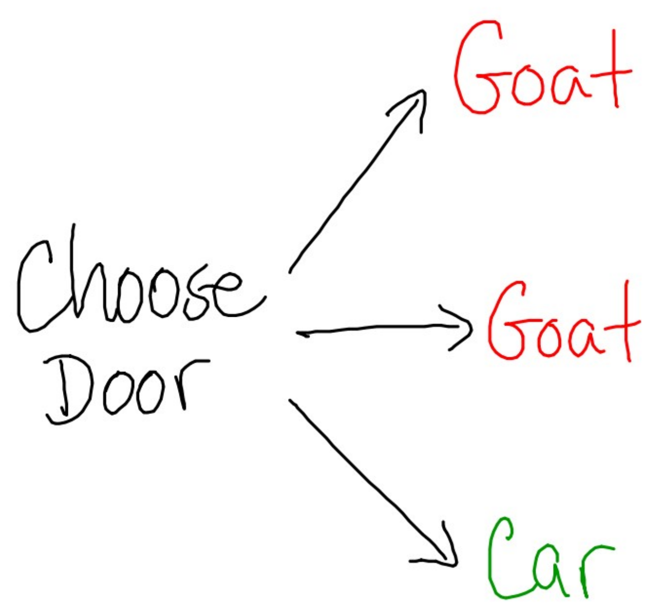
3. THE MONTY HALL PROBLEM*

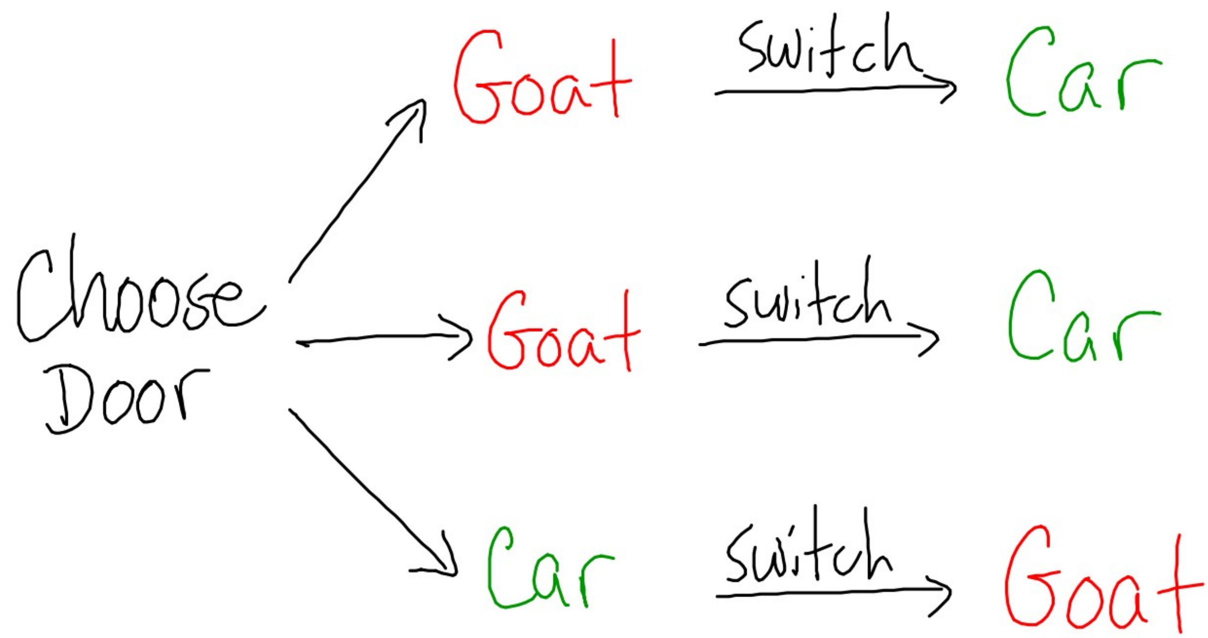
You are on a game show on television. On this game show, the idea is to win a car as a prize. The game show host shows you three doors. He says there is a car behind one of the doors and there are goats behind the other two doors. He asks you to pick a door. You pick a door but the door is not opened. Then the game show host opens one of the doors you didn't pick to show a goat. Then he says that you have one final chance to change your mind before the doors are opened and you get a car or a goat. So he asks if you want to change your mind and pick the other unopened door instead. What should you do?

(After deciding what you should do, see if you are correct by reading the article at: http://math.ucsd.edu/~crypto/Monty/montybg.html)

*Reprinted without permission from the book **The Curious Incident of the Dog in the Night-time** by Mark Haddon

	Goat	Car
Switch	IIII	卌 卌 II
Stick		$\frac{1}{3} = .3333$


Choose Door
Goat
Goat
Car



Choose Door
Goat
switch
Car
Goat
switch
Car
Car
switch
Goat

Exercises, page 309:

5.39 (a) The table below illustrates the possible pair combinations in the sample space. (b) Each of the 16 outcomes has probability $\frac{1}{16}$.

		First Roll			
		1	2	3	4
Second Roll	1	(1,1)	(2,1)	(3,1)	(4,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)

5.40 (a) The sample space is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} (b) Each of the 8 outcomes has probability $\frac{1}{8}$.

$2 \times 2 \times 2 = 8$

5.41 There are four ways to get a sum of 5 from these two dice: (1,4), (2,3), (3,2), (4,1). Each of these has probability $\frac{1}{16}$ so together the probability of getting a sum of 5 is $4\left(\frac{1}{16}\right)=\frac{1}{4}$.

5.42 There are 4 ways to get more heads that tails: HHH, HHT, HTH, THH. Each of these has probability $\frac{1}{8}$ so together the probability of getting more heads than tails is $4\left(\frac{1}{8}\right)=\frac{1}{2}$.

5.43 (a) Legitimate. (b) Not legitimate: the total is more than 1. (c) Legitimate (even if the deck of cards is not!).

5.44 Model 1 is not legitimate because the probabilities have sum $\frac{6}{7}\neq 1$. Model 2 is legitimate. Model 3 is not legitimate because the probabilities have sum $\frac{7}{6}\neq 1$. Model 4 is not legitimate because probabilities cannot be greater than 1 and the sum of the probabilities is more than 1.

5.45 (a) The given probabilities have sum 0.96, so $P(\text{type AB}) = 1 - 0.96 = 0.04$. The sum of all possible outcomes is 1. (b) The probability that the chosen person does not have AB is the sum of all the other probabilities which is 0.96. (c) $P(\text{type O or B}) = 0.49 + 0.20 = 0.69$. } Mutually Exclusive

5.46 (a) The given probabilities sum to 0.91 so $P(\text{other}) = 1-0.91 = 0.09$. (b) $P(\text{non English}) = 1-0.63 = 0.37$. (c) $P(\text{neither English nor French}) = 1-0.63-0.22 = 0.15$.

5.47 (a) The given probabilities have sum 0.72, so this probability must be 0.28. (b) $P(\text{at least a high school education}) = 1 - P(\text{has not finished HS}) = 1-0.13 = 0.87$.

5.48 (a) 35% are currently undergraduates. This makes use of the addition rule of mutually exclusive events because (assuming there are no double majors) "undergraduate students in business" and "undergraduate students in other fields" have no students in common. (b) 80% are not undergraduate business students. This makes use of the complement rule.

5.49 (a) The individuals are the students in the urban school. The variables measured are the children's gender and whether or not they eat breakfast regularly. (b) $P(\text{female}) = \frac{275}{595}$. = .4622

$P(\text{Eats breakfast regularly}) = \frac{300}{595}$. = .5042 $P(\text{Female and eats breakfast regularly}) = \frac{110}{595}$. = .1849

.4622 + .5042 − .1849

$P(\text{Female or eats breakfast regularly}) = \frac{275}{595} + \frac{300}{595} - \frac{110}{595} = \frac{465}{595}$. = .7815

Not Mutually Exclusive

5.50 (a) The individuals are the 100 U.S. senators. The variables being measured are the gender and the political affiliation of the senators. (b) $P(\text{Democrat}) = \frac{(47+13)}{100} = \frac{60}{100} = 0.6$.

$P(\text{Female}) = \frac{(13+4)}{100} = \frac{17}{100} = 0.17$. $P(\text{Female democrat}) = \frac{13}{100} = 0.13$.

6 + .17 − .13

$P(\text{Female or Democrat}) = \frac{60}{100} + \frac{17}{100} - \frac{13}{100} = \frac{64}{100} = 0.64$. > Not Mutually Exclusive

5.51 (a)

	Black	Not Black	**Total**
Even	10	10	**20**
Not Even	8	10	**18**
Total	**18**	**20**	**38**

(b) $P(B) = \frac{18}{38}$; $P(E) = \frac{20}{38}$. (c) The event "B and E" would be that the ball lands in a black, even spot. $P(B \text{ and } E) = \frac{10}{38}$. (d) The event "B or E" would be that the ball lands in either a black spot, or an even spot, but it could be that it lands in both. If we added the separate probabilities for B and E we would double count the probability that it lands in a black, even spot. $P(B \text{ or } E) = \frac{18}{38} + \frac{20}{38} - \frac{10}{38} = \frac{28}{38}$.

5.52 (a)

	Jack	Not a Jack	**Total**
Red Card	2	24	**26**
Black Card	2	24	**26**
Total	**4**	**48**	**52**

(b) $P(J) = \frac{4}{52}$; $P(R) = \frac{26}{52}$. (c) The event "J and R" would be that we deal a red jack. $P(J \text{ and } R) = \frac{2}{52}$. (d) The event "J or R" would be that the card was either a jack, a red card, or both. If we added the separate probabilities for J and R we would double count the probability that the card is both. $P(J \text{ or } R) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$.

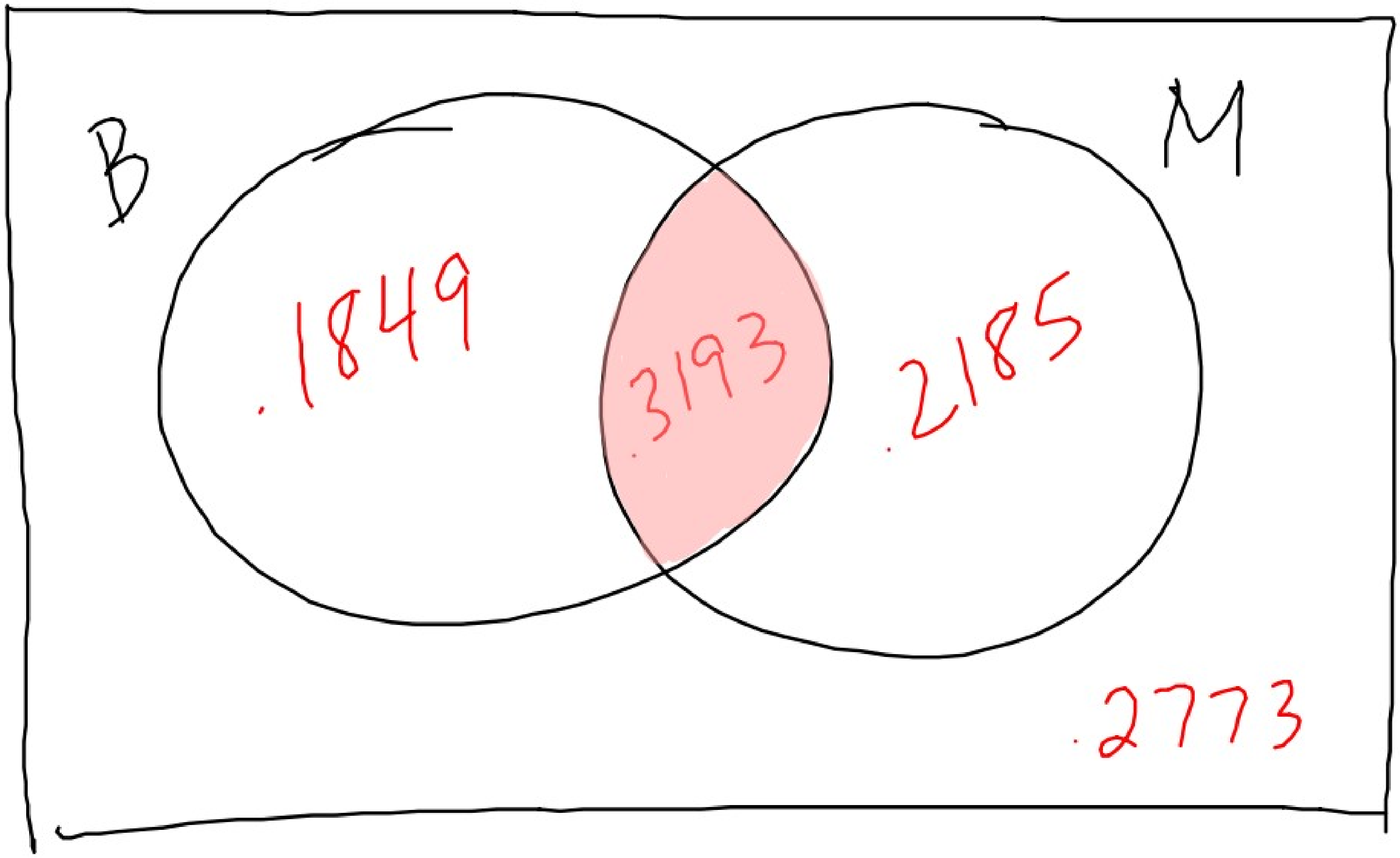
53a) B = eats breakfast regularly

M = Male

$$P(B) = \frac{300}{595} = .5042$$

$$P(M) = \frac{320}{595} = .5378$$

$$P(\text{Both}) = \frac{190}{595} = .3193$$

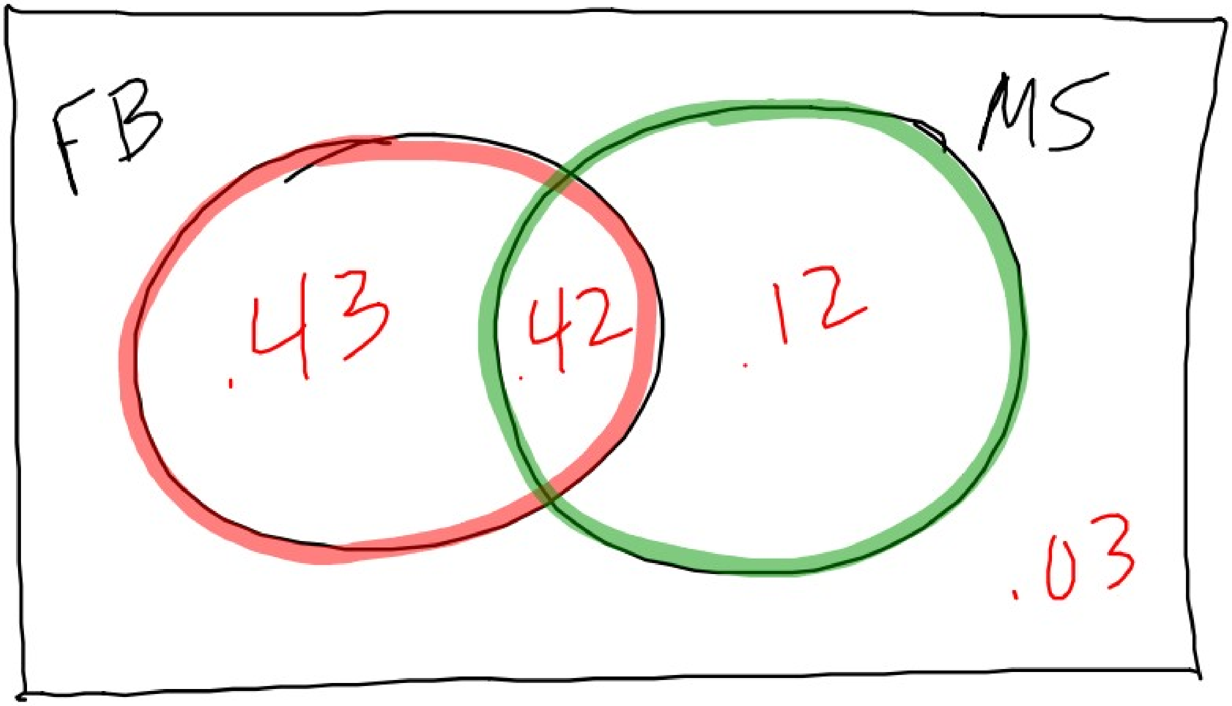


b) $P(B \cup M) = P(B \text{ or } M) = .7227$

c) $P(B^c \cap M^c)$

$= P(\text{No Breakfast and Not Male}) = .2773$

55b)



c) $P(FB \text{ or } MS) = P(FB \cup MS)$

d) $P(FB \cup MS) = .97$

AP STATISTICS

(Section 5.2)

Name(s) ______________________________

1. Suppose you toss one coin and then roll one six-sided die.

a. List the outcomes in the sample space

$S = \{H1\ H2\ H3\ H4\ H5\ H6\ T1\ T2\ T3\ T4\ T5\ T6\}$

b. Find the probability of getting a head

$P(\text{Head}) = \frac{6}{12} = \boxed{.5}$

c. Find the probability of getting a 1, 2 or 3 on the die

$P(1 \text{ or } 2 \text{ or } 3) = \frac{6}{12} = \boxed{.5}$

d. Find the probability of getting a head or a five

$P(\text{Head or } 5) = P(\text{Head}) + P(5) - P(\text{Head and } 5)$

$= \frac{6}{12} + \frac{2}{12} - \frac{1}{12} = \frac{7}{12} = \boxed{.5833}$

2. Below is data on students in a statistics class:

	Juniors	Seniors	
Males	12	8	20
Females	10	6	16
	22	14	36

a. Find the probability of randomly selecting a junior or a female

$P(\text{Junior or Female}) = P(\text{Junior}) + P(\text{Female}) - P(\text{Junior and Female})$

$= \frac{22}{36} + \frac{16}{36} - \frac{10}{36} = \frac{28}{36} = \boxed{.7777}$

b. Find the probability of randomly selecting a student who is not a junior male

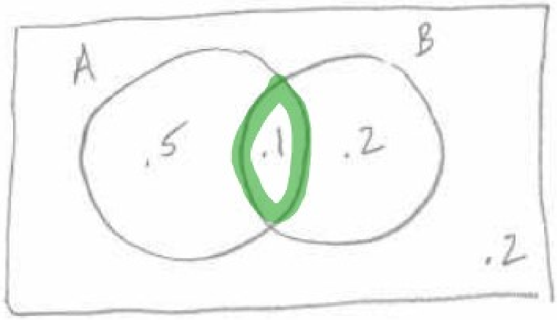
$P(\text{Not Junior Male}) = 1 - P(\text{Junior Male})$

$= 1 - \frac{12}{36}$

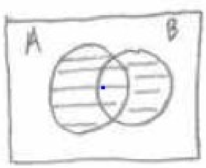
$= \boxed{.6666}$

3. Consolidated Builders has bid on two large construction contracts. The company president believes that the probability of winning the first contract (event A) is 0.6, that the probability of winning the second contract (event B) is 0.3 and that the probability of winning both contracts is 0.1.

a. Construct a Venn diagram that summarizes what you know about events A and B:

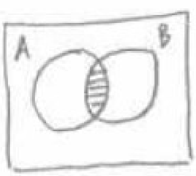


b. Calculate the following probabilities:



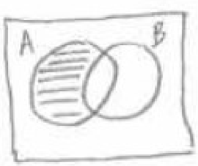
i. $P(A \cup B)$

$= P(A \text{ or } B) = \boxed{.8}$



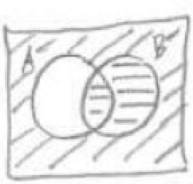
ii. $P(A \cap B)$

$= P(A \text{ and } B) = \boxed{.1}$



iii. $P(A \cap B^C)$

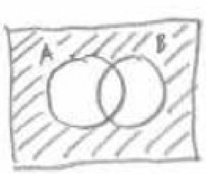
$= P(A \text{ and Not } B) = \boxed{.5}$



iv. $P(A^C \cup B)$

$= P(\text{Not } A \text{ or } B)$

$= P(\text{Not } A) + P(B) - P(\text{Not } A \text{ and } B) = .4 + .3 - .2 = \boxed{.5}$



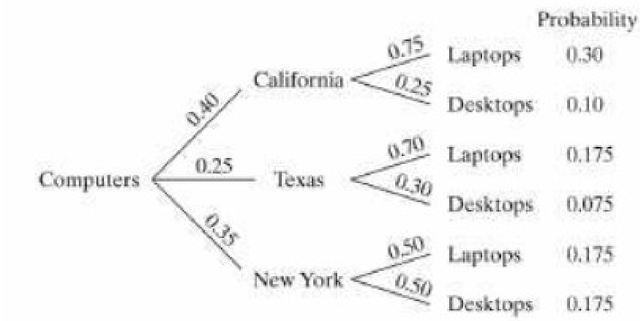
v. $P(A^C \cap B^C)$

$= P(\text{Not } A \text{ and Not } B) = \boxed{.2}$

$\frac{24}{28}=\frac{6}{7}$ of the students are right-handed. And, among the women, $\frac{18}{21}=\frac{6}{7}$ are right-handed. So $P(\text{right-handed})=P(\text{right-handed} \mid \text{female})$.

Check Your Understanding, page 321:

1.



2. $P(\text{laptop})=P(\text{laptop} \cap \text{CA})+P(\text{laptop} \cap \text{TX})+P(\text{laptop} \cap \text{NY})=0.30+0.175+0.175=0.65$.

Check Your Understanding, page 323:

1. $P(\text{one returned safely})=1-P(\text{one was lost})=1-0.05=0.95$. So, if there are 20 missions and whether the bomber returned safely or not is independent for each mission, then $P(\text{safe return on all 20 missions})=P(\text{1st safe}) * P(\text{2nd safe}) * \ldots * P(\text{20th safe})=0.95^{20}=0.3585$.

2. No, we cannot conclude that 2.4% of adults 55 or older are college students. Whether one is a college student and one's age are not independent events. Far more younger people are college students than older people.

Exercises, page 329:

5.63 (a) $P(\text{almost certain} \mid \text{M})=\frac{597}{2459}=0.2428$. (b) $P(\text{F} \mid \text{Some chance})=\frac{426}{712}=0.5983$.

5.64 (a) $P(\text{survived} \mid \text{first class})=\frac{197}{197+122}=\frac{197}{319}=0.6176$. (b) $P(\text{third class} \mid \text{survived})=\frac{151}{197+94+151}=\frac{151}{442}=0.3416$.

5.65 (a) $P(\text{good chance} \mid \text{F})=\frac{663}{2367}=0.2801$. (b) $P(\text{good chance})=\frac{1421}{4826}=0.2944$. (c) The events "a good chance" and "female" are not independent since the two probabilities in parts (a) and (b) are not the same.

5.66 (a) $P(\text{survived} \mid \text{second class})=\frac{94}{94+167}=\boxed{\frac{94}{261}}=0.3602$. (b) $P(\text{survived})=\frac{197+94+151}{197+122+94+167+151+476}=\boxed{\frac{442}{1207}}=0.3662$. (c) The events "survived" and "second class" are not independent since the two probabilities in parts (a) and (b) are not the same.

$P(B|A) \neq P(B)$

5.67 (a) $P(\mathrm{D} \mid \mathrm{F})=\frac{13}{13+4}=\frac{13}{17}=0.7647$. This means that 76.47% of the females are democrats. (b) $P(\mathrm{F} \mid \mathrm{D})=\frac{13}{47+13}=\frac{13}{60}=0.2167$. This means that 21.67% of the democrats are female.

5.68 (a) $P(\mathrm{B} \mid \mathrm{M})=\frac{190}{320}=0.5938$. This means that 59.38% of the males eat breakfast. (b) $P(\mathrm{M} \mid \mathrm{B})=\frac{190}{300}=0.6333$. This means that 63.33% of those who eat breakfast are males.

5.69 $P(\mathrm{D})=\frac{60}{100}=0.60$. From question 5.67 we saw that $P(\mathrm{D} \mid \mathrm{F})=0.7647$. Since these two probabilities are not the same, D and F are not independent. $P(D|F) \neq P(D)$

5.70 $P(\mathrm{B})=\frac{300}{595}=0.504$. From question 5.68 we saw that $P(\mathrm{B} \mid \mathrm{M})=0.5938$. Since these two probabilities are not the same, B and M are not independent.

5.71 (a) $P(\text{is studying other than English})=1-P(\text{none})=1-0.59=0.41$. (b) $P(\text{Spanish} \mid \text{other than English})=\frac{0.26}{0.41}=0.6341$.

$$\frac{P(\text{At least } 50{,}000 \text{ and at least } 100{,}000)}{P(\text{At least } 50{,}000)}$$

5.72 (a) $P(\$50{,}000 \text{ or more})=0.215+0.100+0.006=0.321$. (b) $P(\text{at least } \$100{,}000 \mid \text{at least } \$50{,}000)=\frac{0.106}{0.321}=0.3302$.

5.73 $P(\mathrm{B})<P(\mathrm{B} \mid \mathrm{T})<P(\mathrm{T})<P(\mathrm{T} \mid \mathrm{B})$. There are very few professional basketball players, so $P(\mathrm{B})$ should be the smallest probability. It's much more likely to be over 6 feet tall than it is to be a professional basketball player if you're over 6 feet tall. Lastly, if you are a professional basketball player, it is quite likely that you are tall (larger than the probability that a randomly selected individual is over 6 feet tall).

5.74 Answers may vary. One possible answer is $P(\mathrm{T})<P(\mathrm{T} \mid \mathrm{A})<P(\mathrm{A})<P(\mathrm{A} \mid \mathrm{T})$. Nearly everyone who's career is teaching has a college degree so $P(\mathrm{A} \mid \mathrm{T})$ will have the largest probability (close to 1). And more of the general population will have college degrees than will be teachers, so $P(\mathrm{T})<P(\mathrm{A})$. The final question is where to put $P(\mathrm{T} \mid \mathrm{A})$. Here we have assumed that the proportion of teachers among the college educated is smaller than the proportion of college educated among all people.

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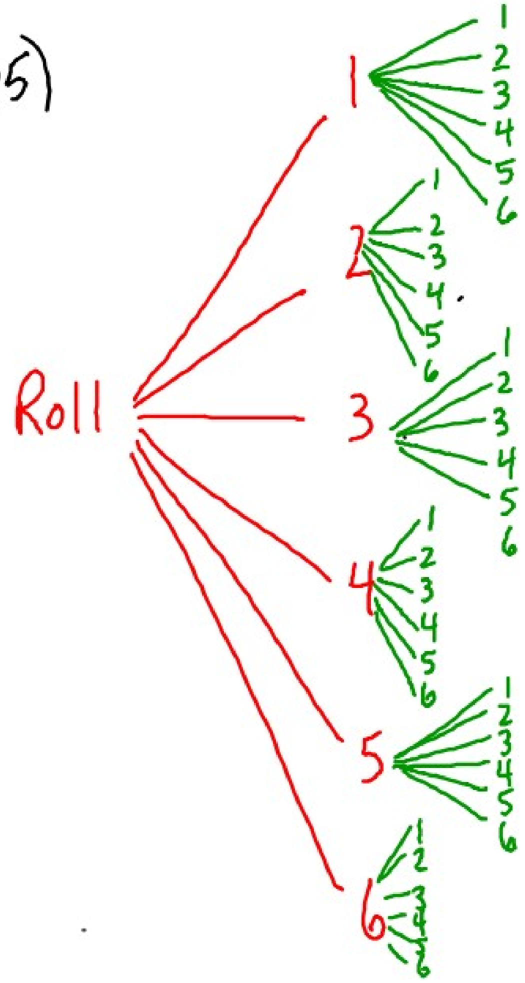
5.75 There are 36 different possible outcomes of the two dice: (1,1), (1,2),…,(6,6). Let's assume that the second die is the green die. There are then six ways for the green die to show a 4: (1,4), (2,4), (3,4), (4,4), (5,4), (6,4). Of those, there is only one way to get a sum of 7, so $P(\text{sum of 7} \mid \text{green is 4})=\frac{1}{6}=0.1667$. Overall, there are 6 ways to get a seven: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). So

75)



$$P(\text{Sum of } 7) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{Sum of } 7 \mid \text{Green Die Shows } 4) = \frac{1}{6}$$

Independent

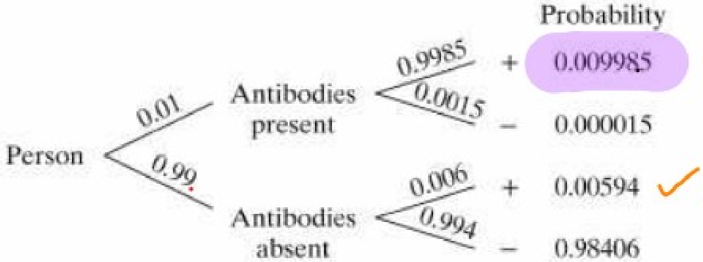
From the tree diagram, add the probabilities for all branches that end in lactose intolerant. This leads to $P(\text{lactose intolerant}) = 0.123 + 0.098 + 0.036 = 0.257$. (b) We are looking for

$$P(\text{Asian} \mid \text{lactose intolerant}) = \frac{P(\text{Asian} \cap \text{lactose intolerant})}{P(\text{lactose intolerate})} = \frac{0.036}{0.257} = 0.1401.$$

5.98 (a) Add the probabilities for all branches that end in a contribution. This involves multiplying along the branches.
$P(\text{contribute}) = (0.5)(0.4)(0.8) + (0.3)(0.3)(0.6) + (0.2)(0.1)(0.5) = 0.16 + 0.054 + 0.01 = 0.224.$

(b) We are looking for $P(\text{recent donor} \mid \text{contribute}) = \dfrac{P(\text{recent donor} \cap \text{contribute})}{P(\text{contribute})} = \dfrac{0.16}{0.224} = 0.7143.$

5.99 (a)



(b) Add the probabilities for all branches that end in a positive test.
$P(\text{positive}) = 0.009985 + 0.00594 = 0.015925.$ (c)

and

$$P(\text{antibody} \mid \text{positive}) = \frac{P(\text{antibody} \cap \text{positive})}{P(\text{positive})} = \frac{0.009985}{0.015925} = 0.6270.$$

5.100 (a) A false-positive means that the technician identified someone as having the disease when, in fact, they were healthy. This happened in 50 out of 750 healthy people so the false-positive rate was $\frac{50}{750} = 0.0667$. A false-negative means that the technician identified someone as being healthy when, in fact, they had the disease. This happened in 10 out of 250 patients so the false-negative rate was $\frac{10}{250} = 0.04$. (b) We are looking for $P(\text{had disease} \mid \text{postive result}) = \dfrac{P(\text{had disease} \cap \text{positive result})}{P(\text{positive result})}$.
Out of the 1000 people tested, there were $240 + 50 = 290$ positive results, so $P(\text{positive result}) = \frac{290}{1000} = 0.29$. There were 240 people who had the disease and got a positive result so $P(\text{had disease} \cap \text{positive result}) = \frac{240}{1000} = 0.24$. This gives

$$P(\text{had disease} \mid \text{postive result}) = \frac{0.24}{0.29} = 0.8276.$$

Probability

1. AARDVARK

The letters in the word AARDVARK are printed on square pieces of cardboard (same size squares) with letter per card The eight letters are then placed in a hat and one letter card is randomly chosen from the hat.

a) Complete the table:

Outcomes	A	R	D	V	K
Probability	$\frac{3}{8}=.375$	$\frac{2}{8}=.25$	$\frac{1}{8}=.125$	$\frac{1}{8}=.125$	$\frac{1}{8}=.125$

Consider the following events:

V: the letter chosen is a vowel
F: The letter chosen falls in the first half of the alphabet (A – M)

Determine the following probabilities:

b) P(V) $\frac{3}{8}=.375$

c) P(F) $\frac{5}{8}=.625$

d) P(V or F) $\frac{5}{8}=.625$

e) $P(F^c)$ $1-\frac{5}{8}=\frac{3}{8}=.375$

f) Determine if the events V and F are independent:

$P(V)=P(V|F)?$ or $P(F)=P(F|V)?$

$\frac{3}{8}\neq\frac{3}{5}$ $\frac{5}{8}\neq\frac{3}{3}$

Not Indpd Events

2. AP FREE-RESPONSE QUESTION (1999)

Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on it.

Suppose you are the first player and you want to win the game. Which die would you select and why?

$P(A\text{ Wins}) = P(A\text{ Gets }9)\text{ and }P(B\text{ Gets }3)$

$=\frac{4}{6}\cdot\frac{4}{6}=\boxed{\frac{4}{9}}$

$P(B\text{ Wins}) = 1-\frac{4}{9}=\boxed{\frac{5}{9}}$

You should select die B because the odds of winning are 5 out of 9