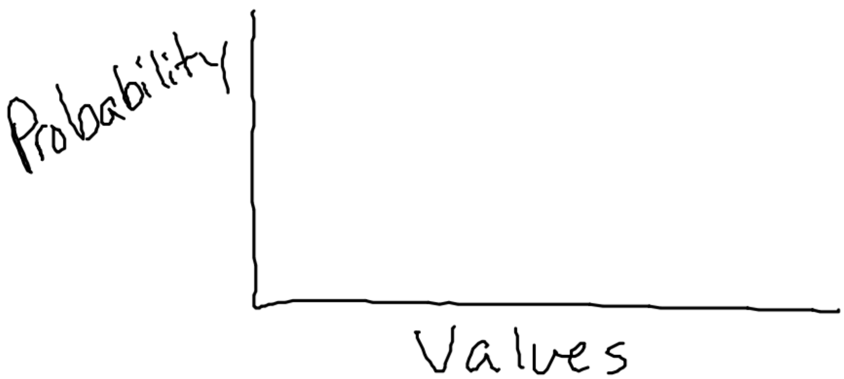
Sec 7.1

Probability Distributions



Types of Probability Distributions

- Normal Distribution (Ch 2) 
- Prob Dist of Random Variables (Ch 7)
- Binomial Distributions
- Geometric Distributions

} Ch 8

Random Variable (X)

A variable whose value is a probability of a random phenomena

Discrete → Probability Distribution

X = the number of __

Continuous → Density Curves

X = the amount of __

Ex Discrete Random Variable

The probabilities that a BP customer selects 1, 2, 3, 4 or 5 items are:

X	1	2	3	4	5
P(X)	.32	.12	.23	.18	.15

= 1.00 ✓

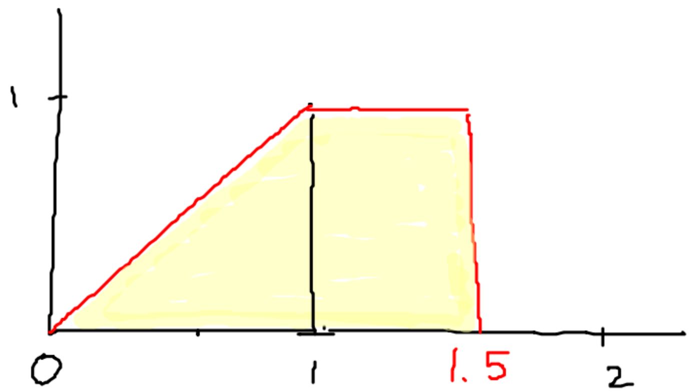
$P(X > 3.5) = P(X = 4 \text{ or } 5) = .33$

$P(1.0 < X < 3.0) = P(X = 2) = .12$

$P(X < 5) = P(X \neq 5) = 1 - .15 = .85$

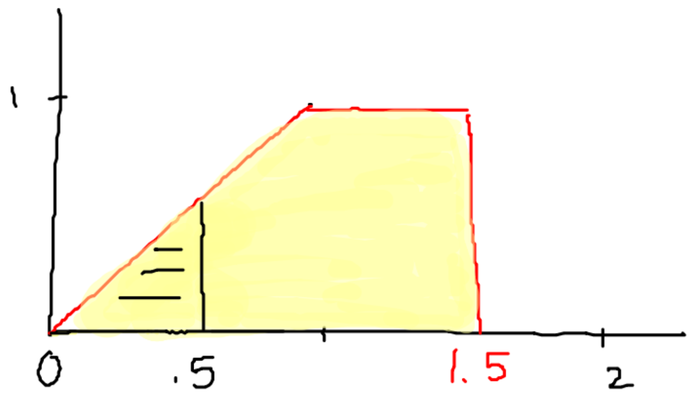
Ex Continuous Random Variable

A probability density function is comprised of 2 line segments. The first segment begins at $(0,0)$ and goes to $(1,1)$. The second segment goes from $(1,1)$ to $(1.5,1)$.



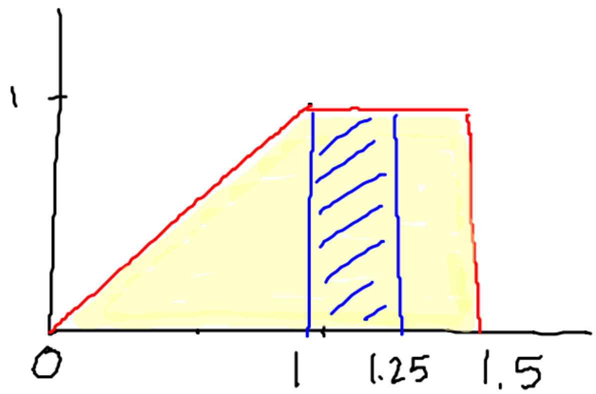
1) Verify legitimate density curve $(A = 1)$

$A = .5 + .5 = 1$ ✓



2) $P(0 < X \leq .5)$

$A = \frac{1}{2}bh = \frac{1}{2}(.5)(.5) = .125$



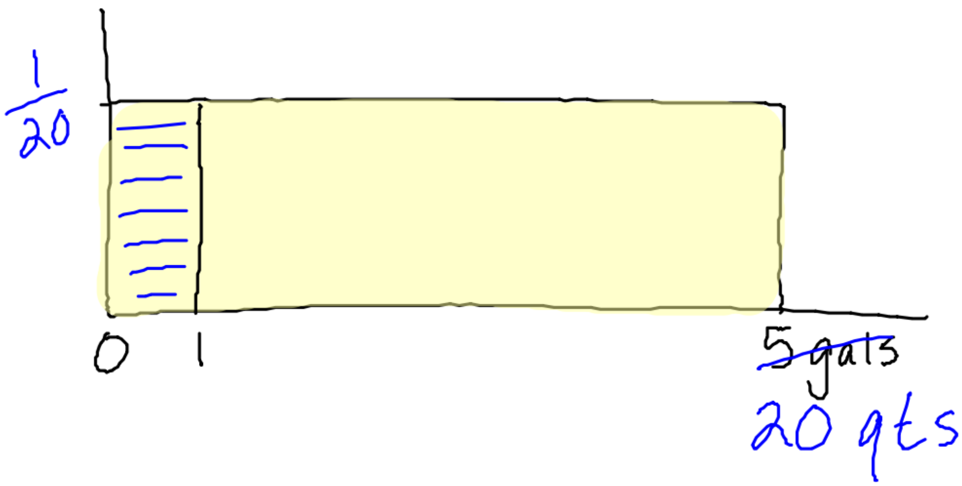
3) $P(1 < X < 1.25)$

$A = (.25)(1) = .25$

4) $P(X = 1) = 0$ } No Area Above A Point!

Ex Continuous Random Variable

Assume the amount of ice cream in an open 5-gallon container is uniformly distributed. What is the probability that the sales person will have to open a new 5-gallon container when you order a quart of ice cream?



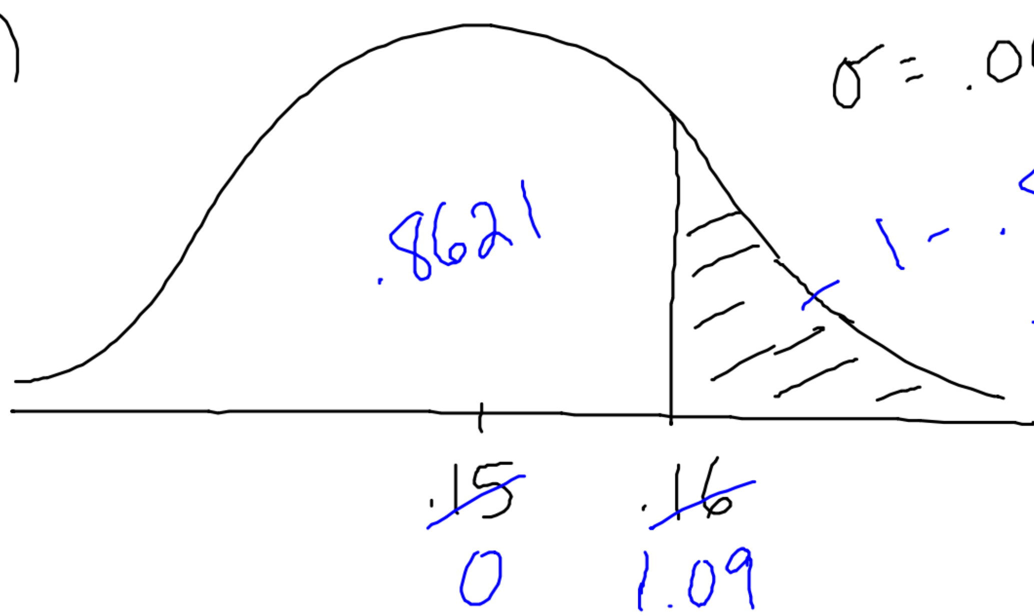
$$P(X < 1) = \frac{1}{20} = .05$$

Notes

- Ignore $<$ vs $\leq$ ($>$ vs $\geq$) for continuos random variables
- $X + X \neq 2X$

7.20a)

$\sigma = .0092$



$1 - .8621 = .1379$

normalcdf(.16, 100, .15, .0092) ≈ .1385

7.20b)

normalcdf (.14, .16, .15, .0092) = .7242

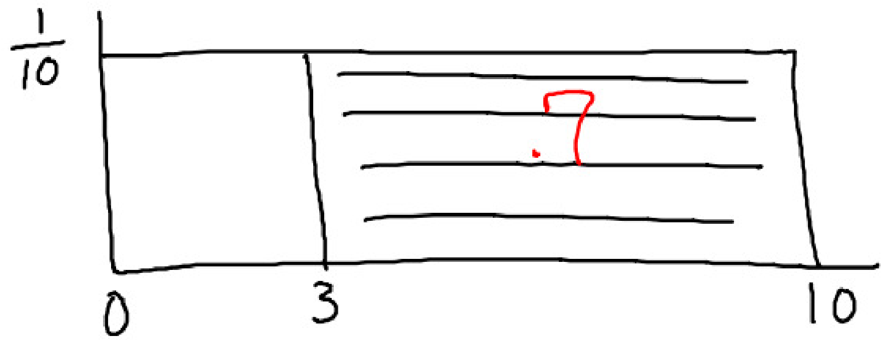
[.8621 - .1379 = .7242]

NOTES QUIZ
(Section 7.1)

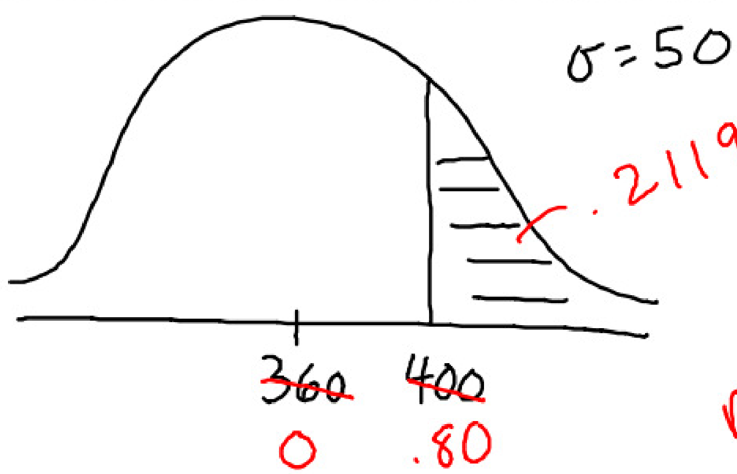
Name ____

CHOOSE ONE OF THE FOLLOWING; SHOW ALL WORK:

1. During rush hour, subway trains run every 10 minutes, If you arrive on the platform at a random time, what is the probability that you will have to wait more than 3 minutes for the next train?



2. Let the random variable X represent the profit made on a randomly selected day by a certain store. Assuming X is normal with a mean of \$360 and standard deviation of \$50, find P(X > \$400):



normalcdf(400, 1000, 360, 50)

Sec 7.2

Mean of Random Variable (μ_X)

Ex Size of Households (Ex 7.10)

# Inhabitants	1	2	3	4	5	6	7
Probability	.25	.32	.17	.15	.07	.03	.01

$$\mu X = 1(.25) + 2(.32) + 3(.17) + \cdots + 7(.01) = 2.6$$

Expected Value

$$E(X) = \mu_X = \sum x_i p_i$$

Ex Game of Chance

If a player rolls 2 dice and gets a sum of 2 or 12, s/he wins \$20. If s/he gets a sum of 7, the player wins \$5. The cost to play is \$3.

Find the expected payout for the game.

X = the payout

X	\$0	\$5	\$20
$P(x)$	$28/36$	$6/36$	$2/36$

$$\mu_X = 0\left(28/36\right) + 5\left(6/36\right) + 20\left(2/36\right) = \$1.94$$

Is the game fair?

Variance Review (Ch 1)

$$s^2 = \frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{n-1}$$

Standard Deviation

$$s = \sqrt{\text{Variance}}$$

Variance of Random Variables

$$\sigma_X^2 = (X_1 - \mu_X)^2 p_1 + (X_2 - \mu_X)^2 p_2 + \cdots + (X_n - \mu_X)^2 p_n$$

$$\mathrm{Var}(X) = \sigma_X^2 = \sum (X_i - \mu_X)^2 p_i$$

$$\sigma_X^2 = (1-2.6)^2(.25) + (2-2.6)^2(.32) + (3-2.6)^2(.17) + (4-2.6)^2(.15) + \cdots + (7-2.6)^2(.01) = 2.02$$

$$\sigma_X = \sqrt{2.02} = 1.42$$

Ch 1 Review

	Data Set	$\bar{x}$		s	
	{2, 5, 8}	5		3	
+4	{6, 9, 12}	9	+4	3	NC
x2	{4, 10, 16}	10	x2	6	x2
x2 +4	{8, 14 20}	14	x2 +4	6	x2

Rules For Random Variables

1) $Y = bX + a$

$$\mu_Y = b\mu_X + a$$

$$\sigma_Y = b\sigma_X$$

$$\left[\sigma^2_Y = b^2\sigma^2_X\right]$$

<u>Ex</u> Let $\mu_x = 3$ $\sigma_x = 1.4$

and $Y = .9X - .2$

$\mu_y = .9(3) - .2 = 2.5$

$\sigma_y = .9(1.4) = 1.26$

<u>Ex</u> Assume $\sigma_X = 20$

Find a and b such that $Y = bX + a$ has a standard deviation of 1.

$$\sigma_Y = 1$$

$$b\sigma_X = 1$$

$$20b = 1$$

$$b = \frac{1}{20} \longrightarrow a = \{\text{reals}\}$$

2) $\mu_{x+y} = \mu_x + \mu_y$

x = SAT Math Score ($\mu_x = 625$)

y = SAT Verbal Score ($\mu_y = 590$)

$\downarrow$

$\mu_{x+y} = 625 + 590 = 1215$

3) $\sigma^2_{x+y} = \sigma^2_x + \sigma^2_y$

BUT $\sigma_{x+y} \neq \sigma_x + \sigma_y$

Ex $\sigma_x = 10$, $\sigma_y = 8$, find σ_{x+y}

$\sigma^2_x + \sigma^2_y = 100 + 64 = 164$

$\sigma_{x+y} = \sqrt{164} = 12.81$

Ex

HS Relay Team	Mean (Mile)	Stand Dev
Runner 1	4.9 min	.15
2	4.7	.16
3	4.5	.14
4	4.8	.15

1) What is the team's mean time?

$$\mu_T = 4.9 + 4.7 + 4.5 + 4.8 = 18.9 \text{ min}$$

2) What is the team's standard deviation?

$$\sigma^2_T = (.15)^2 + (.16)^2 + (.14)^2 + (.15)^2 = .0902$$

$$\sigma_T = \sqrt{.0902} = .3003$$