Sec 8.1

Inference

It is so hot in here... would someone please open a ________

reasonable answer based on context (data)

Statistical Inference

Sample Data

↓

Sample Statistic ($\bar{x}$ or $\hat{p}$) } Point Estimator

↓

Make reasonable conclusion about population parameter

↙ ↘

Confidence Interval | Hypothesis Test

Confidence Intervals

- Used to "bet" on an interval which captures a population proportion or population mean

- Does NOT actually find a parameter

MOE

CI = point estimator ± (critical value)(standard deviation)

↑ determined by Confidence Level

90% Confidence Level means that 90% of all samples would create an interval that captures the true population parameter (See P.474)

Interpreting A CI

Gallup survey asked 439 teens (13-17) whether they thought young people should wait until marriage to have sex

$$95\%\ CI = .56 \pm .04 = (.52, .60)$$

This Means:

"We are 95% confident that the interval from .52 to .60 captures the actual proportion of all teens who think young people should wait until marriage to have sex"

This Does NOT Mean:

- There is a 95% chance that the interval from .52 to .60 captures the actual proportion..."

- Either the interval captures the population proportion or it doesn't!

Decreasing MOE (CI = statistic ± MOE)

1) Decrease Confidence Level (95% → 80%)

Less confident → More Precise

2) Increase Sample Size

Decreases standard deviation

Conditions Required For CIs

1) Random (SRS or Randomized Experiments)

2) Normal (Sampling Distribution)

3) Independent ($N > 10n$)

Sec 8.2

Estimating A Population Proportion

1) Choose A Confidence Level

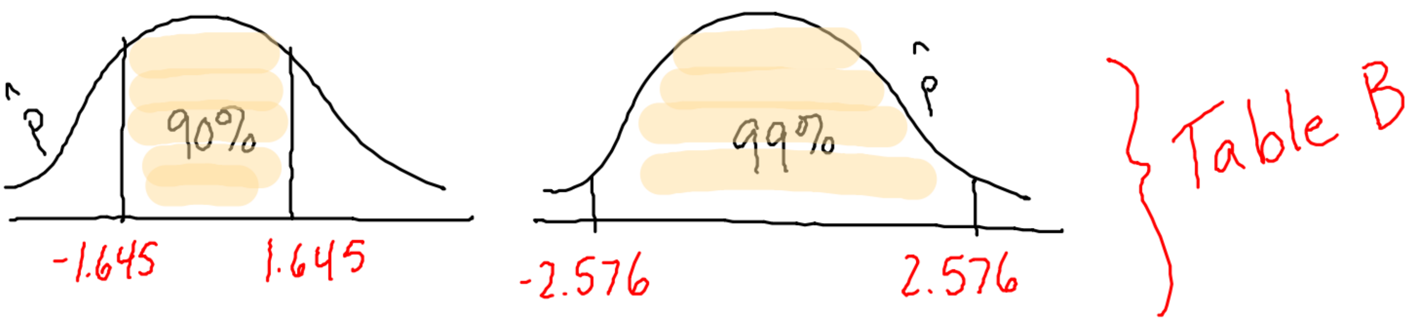


Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

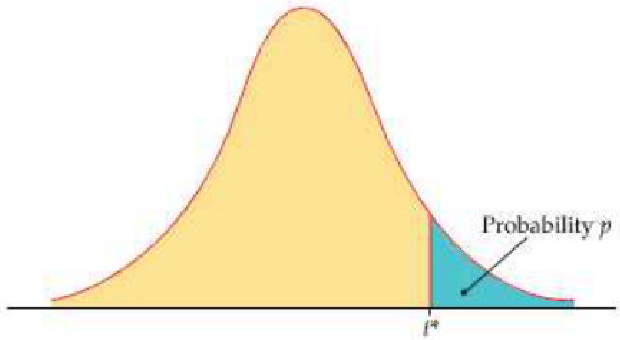


TABLE D

t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

2) Follow Steps

P State population parameter of interest

A State assumptions required

- ✓ Random
- ✓ Normal
- ✓ Independent

I Construct Interval ← standard error

$$CI = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

S State conclusion <u>in</u> <u>context</u>

1- PROPORTION Z-INTERVAL

This test is used to construct a confidence interval for a population proportion (p) using a sample proportion ($\hat{p}$)

The Gallup Youth Survey asked a random sample of 439 U.S. teens aged 13 to 17 whether they thought young people should wait to have sex until marriage. Of the sample, 246 said "Yes".

Construct and interpret a 95% confidence interval for the proportion of all teens who would say "Yes" to this question.

P) IDENTIFY POPULATION PARAMETER:

p = proportion of US teens (13-17) who think young people should wait until marriage to have sex

A) VERIFY CONDITIONS REQUIRED FOR TEST:

✓ a) Random:

Random sample was used

b) Normal:

.5604

$n\hat{p} = (439)(246/439) = 246 \geq 10$ ✓

$n(1-\hat{p}) = (439)(193/439) = 193 \geq 10$ ✓

.4396

c) Independent:

$N > 10(439) = 4,390$ US teens ✓

T) CONSTRUCT INTERVAL

a) USE *t* DISTRIBUTION TABLE:

$$95\%\ CI = \hat{p} \pm z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= .56 \pm (1.96)\sqrt{\frac{(.56)(.44)}{439}}$$

$$= .56 \pm .046$$

$$= (.514, .606)$$

b) USE CALCULATOR:

STAT → TESTS → 1-Prop Z Int = (.51, .61)

↑ X = # successes

S) STATE CONCLUSION:

We are 95% confident that the interval from .51 to .61 captures the actual proportion of all US teens (13-17) who believe young people should wait until marriage to have sex

Sample Size vs Desired MOE

A company wants to construct a 95% CI with a MOE $\leq$ 3%. How large should the sample be?

$$CI = \hat{p} \pm \underbrace{z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}_{\text{MOE}}$$

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq .03$$

$$1.96 \sqrt{\frac{(.50)(.50)}{n}} \leq .03$$

$$\frac{(1.96)(.50)}{\sqrt{n}} \leq \frac{.03}{1}$$

$$.98 \leq .03\sqrt{n}$$

$$32.67 \leq \sqrt{n}$$

$$1067.1 \leq n$$

The company would need 1068+ people

Sec 8.3

Estimating A Population Mean

1) Choose a Confidence Level

2) Follow Steps

P Identify population mean of interest (μ)

A State assumptions required

Random

Normal Sampling Distribution

Normal population ?

$n > 30$ (Central Limit Thm) ?

$n < 30$, check data ?

Independent

I Construct Interval

$$CI = \bar{X} \pm z^* \frac{\sigma}{\sqrt{n}}$$ when σ known

$$CI = \bar{X} \pm t^* \frac{s}{\sqrt{n}}$$ when σ unknown

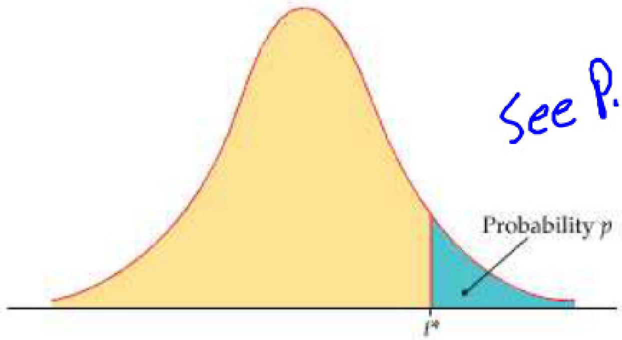
↑ uses degrees of freedom $(n-1)$

as $n \to \infty$, $t^* \to z^*$

S State conclusion in context

See P. 505

Table entry for *p* and *C* is the critical value t^* with probability *p* lying to its right and probability *C* lying between $-t^*$ and t^*.



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26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
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60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

Z and T INTERVALS

These tests are used to construct a confidence interval for a population mean (μ).

When σ is known, use a Z-Interval; when σ is unknown, use a T-Interval.

> The tension readings in millivolts (mV) from a random sample of 20 screens from a single day's production are as follows:
>
> 269.5 297.0 269.6 283.3 304.8 280.4 233.5 257.4 317.5 327.4
> 264.7 307.7 310.0 343.3 328.1 342.6 338.8 340.1 374.6 336.1
>
> **Construct and interpret a 90% confidence interval for the mean tension of all the screens produced on this day.**

P) STATE POPULATION PARAMETER:

μ = average tension reading (mV) of all screens produced on this day

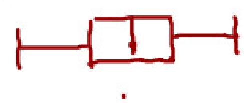
A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) ✓Random

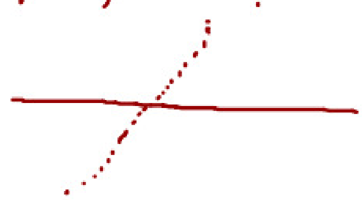
Random Sample used

b) Normal parent ~~population~~ or large ~~sample~~ size or justification for normality

1) Check for outliers (modified box plot)



2) Normal Probability Plot (npp)



c) Independence

Number of screens produced > 20(10) > 200?

I) CONSTRUCT INTERVAL

Since σ is unknown, we will construct a T-Interval:

a) USE *t* DISTRIBUTION TABLE:

i) Determine mean ($\bar{x}$) and standard deviation (s) > 1-Var Stats

$\bar{X} = 306.32$, $S = 36.21$

ii) Determine t^* using (20-1) degrees of freedom

$t^* = 1.729$ (df = 19, 90% CI)

iii) Construct confidence interval

$$CI = \bar{X} \pm t^* \frac{S}{\sqrt{n}}$$

$$= 306.32 \pm (1.729) \boxed{\frac{36.21}{\sqrt{20}}}$$ standard error

$$= (292.32, 320.32)$$

b) USE CALCULATOR

STAT → TESTS → TInterval → (292.32, 320.32)

S) STATE CONCLUSION:

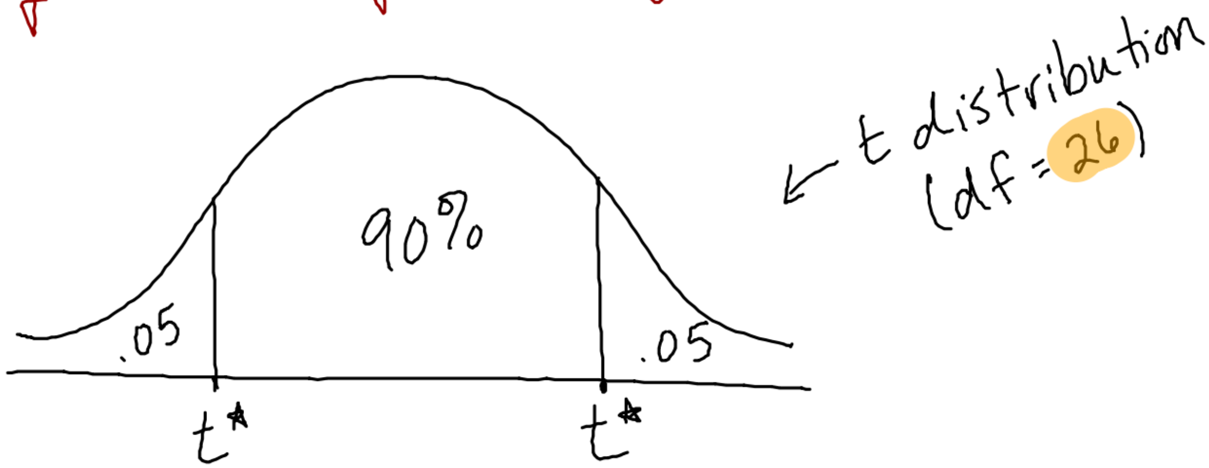
We are 90% confident that the interval from 292.32 mV to 320.32 mV captures the true population mean of all screens produced that day

Robust Procedures

- t intervals are robust (fairly accurate) even if data is nonNormal unless there are outliers

- See P. 513

Finding t^* (Using Technology)



Either:

- DISTR → invT(.05, 26)
- DISTR → invT(.95, 26)

$t^* = 1.70561784$