# Sec 12-3

# (Analyzing Data)

# 1) Measures of Central Tendency

## a) Mean ($\bar{x}$)

- Arithmetic average $\bar{X} = \frac{\sum x_i}{n}$
- Affected by extreme values (outliers)

Ex $2, 4, 6 \rightarrow \bar{X} = 4$

$2, 4, 18 \rightarrow \bar{X} = 8$

## b) Median (M)

- Midpoint of values

Ex 2, 4, 6 → M = 4

2, 4, 6, 8 → M = 5

- Not affected by outliers

Ex 2, 4, 18 M = 4

## c) Mode

- Most frequent value

## 2) Spread (Sec 12-4)

# 3) 5-Number Summary

16 19 24 25 | 25 25 33 34 | 34 37 37 40 | 42 46 49 73

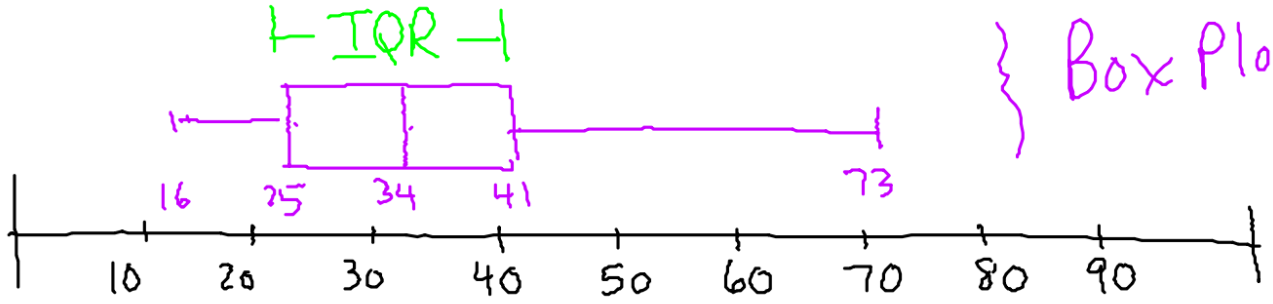
min = 16

$Q_1 = 25$

$M = 34$

$Q_3 = 41$

max = 73




} Box Plot

IQR - Interquartile range ($Q_3 - Q_1$)

# 4) Outliers (IQR Rule)

$Q_1 - 1.5(IQR) > \text{Outlier} > Q_3 + 1.5(IQR)$

$25 - 1.5(16) > \text{Outlier} > 41 + 1.5(16)$

$-1 > \text{Outlier} > 65$

# Percentiles

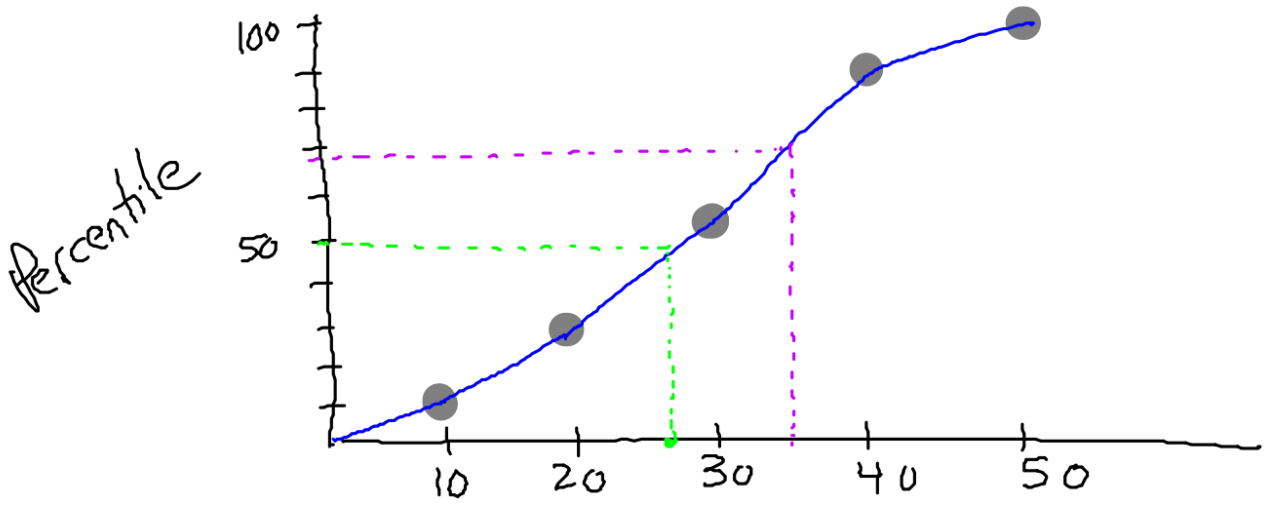
Percent of data that falls below a certain value

Ex PSAT Score → 90th Percentile

90% of students got less than your score

# Ogive (O-Jive)

| Test Score | Frequency | Relative Frequency | Relative Cumulative Frequency |
|---|---|---|---|
| 0 - 9 | 4 | 10% | 10% |
| 10 - 19 | 8 | 20 | 30 |
| 20 - 29 | 10 | 25 | 55 |
| 30 - 39 | 15 | 37.5 | 92.5 |
| 40 - 50 | 3 | 7.5 | 100 |
| | 40 | ✓ 100% | |




27 falls in 50th Percentile

35 is in the 68th Percentile

# Sec 12-4

# (Measuring Spread)

# Measures of Dispersion

1) Range (max - min)

2) IQR $(Q_3 - Q_1)$

↑ Middle 50%

# 3) Standard Deviation (s)

a) $$s = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$$

b) $$s = \sqrt{\frac{n\sum x^2 - \left(\sum x\right)^2}{n(n-1)}}$$

Ex Find standard deviation of $\{2, 4, 6\}$

$\bar{x} = 4$

a) $$s = \sqrt{\frac{(2-4)^2 + (4-4)^2 + (6-4)^2}{3-1}}$$

$$= \sqrt{\frac{4 + 0 + 4}{2}}$$

$$= \sqrt{4}$$

$$= 2$$

$\{2, 4, 6\}$

b) $$s = \sqrt{\frac{3(4+16+36) - (2+4+6)^2}{3(3-1)}}$$

$$= \sqrt{\frac{168 - 144}{6}}$$

$$= \sqrt{4}$$

$$= 2$$

Note

Standard deviation is not resistant (affected by outliers)

# Z - Score

- The number of standard deviations that a value is from the mean
- Generally $-3 \leq z \leq 3$

Ex Find Z-score of 620

$$Z = \frac{x - \bar{x}}{s} = \frac{620 - 500}{100} = 1.2$$

Ex Find Z-score of 345

$$Z = \frac{345 - 500}{100} = -1.55$$

Ex SAT Math Scores $(\bar{X}=500,\ s=100)$


| Z-Score | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|

Sec 12-5

## Population

An entire set of people (or things)

## Parameter

A number (mean $\mu$ or proportion $p$) used to describe a population from a census

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## Sample

Subset of population chosen randomly

## Statistic

A number (mean $\bar{x}$ or proportion $\hat{p}$) used to describe a sample ... used to predict a parameter

## Proportion Property

When a large sample of size $n$ is taken from a large population then the likely range for the population proportion is $\hat{p} \pm \frac{1}{\sqrt{n}}$

↑ Margin of Error (MOE)

Ex In a survey of 2750 airline travelers, 138 say they never check their luggage when they fly. Find the sample proportion, the margin of error and the interval likely to contain the true population proportion.

a) $\hat{p} = \frac{138}{2750} \approx 5\%$

b) $MOE = \frac{1}{\sqrt{2750}} \approx 2\%$

c) $p = (5 \pm 2) = 3\%$ to $7\%$

Ex A poll reports 56% of voters prefer Candidate A with a MOE of 3%. Estimate the number of voters in the poll.

$$MOE = \frac{1}{\sqrt{n}}$$

$$\frac{.03}{1} = \frac{1}{\sqrt{n}}$$

$$.03\sqrt{n} = 1$$

$$\sqrt{n} = \frac{1}{.03} = 33.\overline{3}$$

$$n = 1{,}112$$

Sec 12-6

# Binomial Setting

1) 2 Outcomes (Success / Failure)

2) P(success) = constant

3) Exact number of $n$ items

Ex 5-Choice MC Test With 50 questions:

1) 2 Outcomes (Correct / Incorrect)

2) $P(\text{success}) = \frac{1}{5} = .20$

3) Exact number of questions = $n = 50$

Find the probability that you get exactly 10 questions correct by randomly guessing

$$P(X=k) = {}_nC_k (p)^k (1-p)^{n-k}$$

$$P(X=10) = {}_{50}C_{10} (.20)^{10} (.80)^{40} = .1398$$

Find the probability that you get at least 3 questions correct on a 5-question MC test by guessing

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$= {}_5C_3(.2)^3(.8)^2 + {}_5C_4(.2)^4(.8)^1 + {}_5C_5(.2)^5(.8)^0$$

$$= .0512 + .0064 + .00032$$

$$= .0579$$

Find the probability that you get less than 3 questions correct by randomly guessing.

$$P(X<3) = P(X=2) + P(X=1) + P(X=0)$$

$$= 1 - P(X \geq 3)$$

$$= 1 - .0579$$

$$= .9429$$

A calculator contains 4 batteries. With normal use, each battery has a 90% chance of lasting for one year. Find the probability that all 4 last for one year.

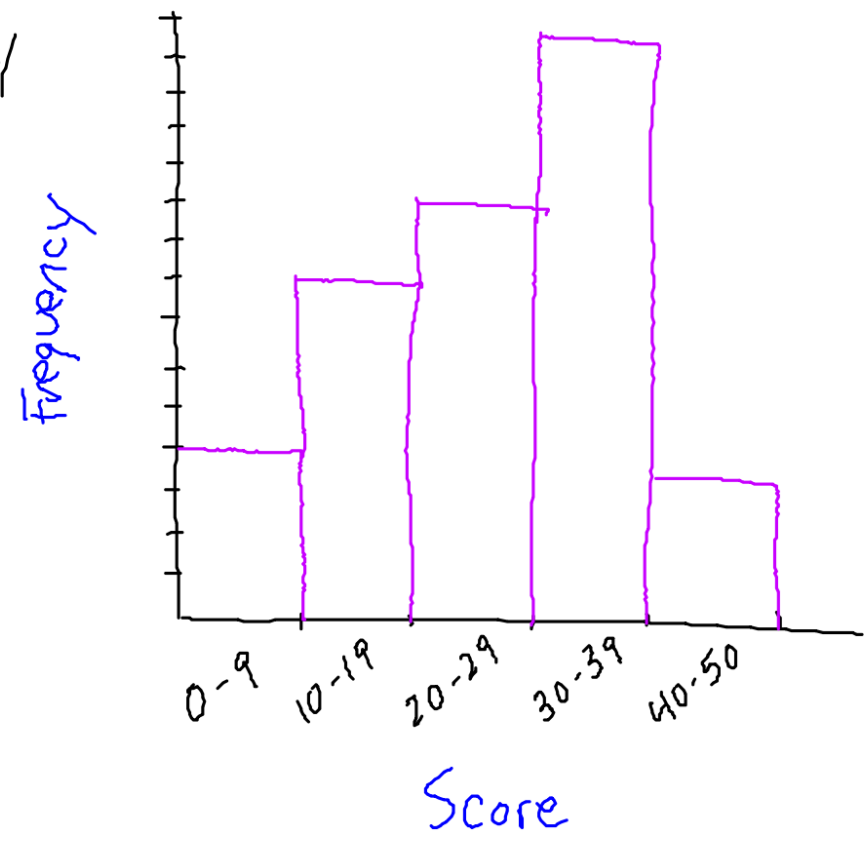
$$P(X=4) = {}_4C_4 (.90)^4 (.10)^0 = .6561$$

Sec 12-7

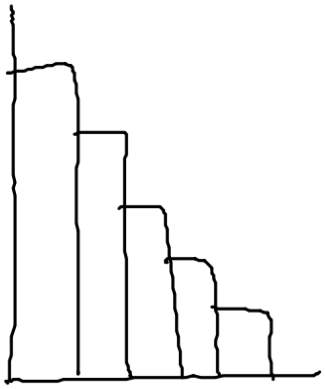
The shape a set of data make is called its distribution

Histogram

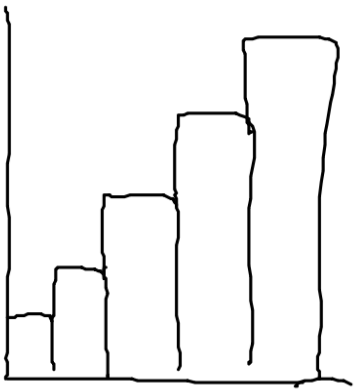
| Score | Frequency |
|---|---|
| 0 - 9 | 4 |
| 10 - 19 | 8 |
| 20 - 29 | 10 |
| 30 - 39 | 15 |
| 40 - 50 | 3 |



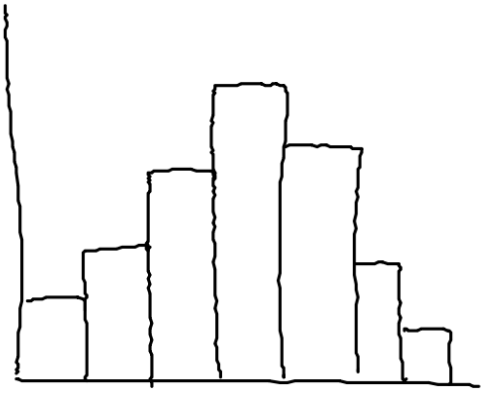
# Shapes



Skewed Right



Skewed Left



★ Symmetric

# Normal Distribution

- Model used to represent symmetric data with a bell-shaped curve
- See Handout (Vocabulary Scores)

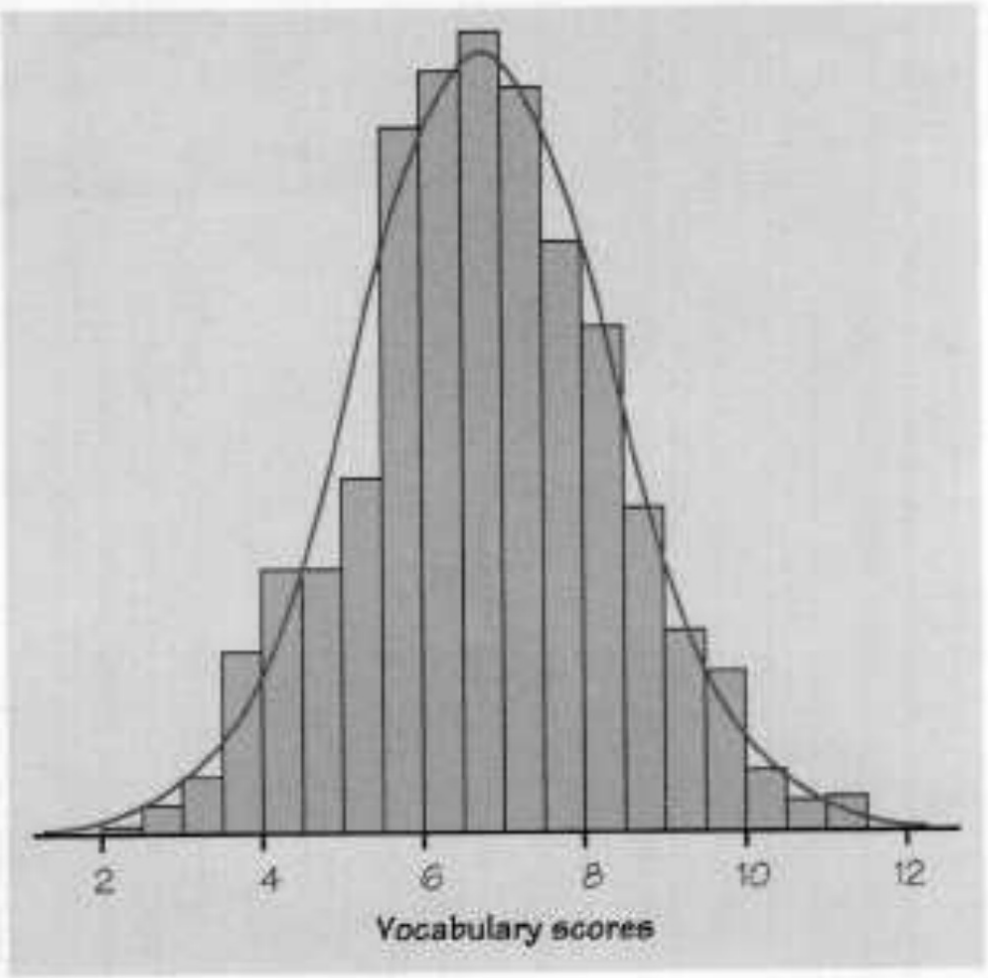



FIGURE 2.2 Histogram of the vocabulary scores of all seventh-grade students in Gary, Indiana. The smooth curve shows the overall shape of the distribution.

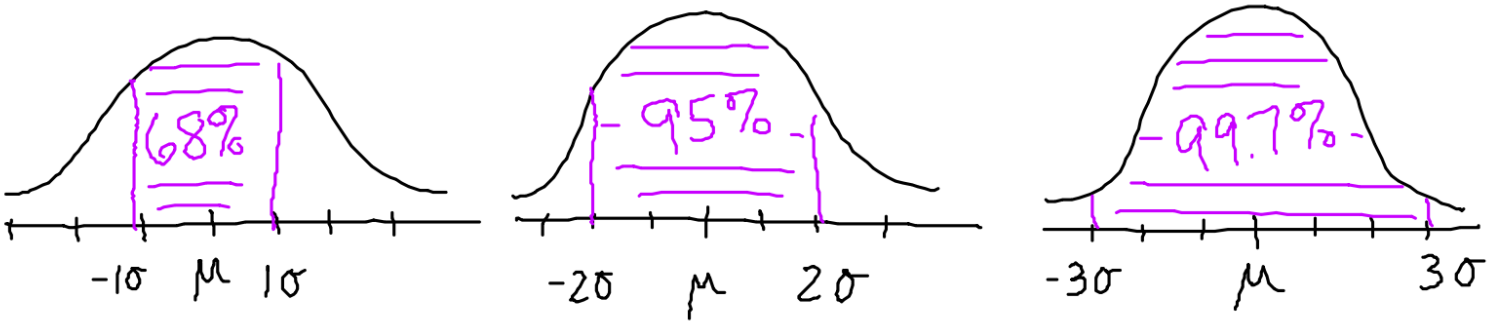
# Notation

$$N(\mu, \sigma)$$

Mean

Standard Deviation

# 68-95-99.7 Rule

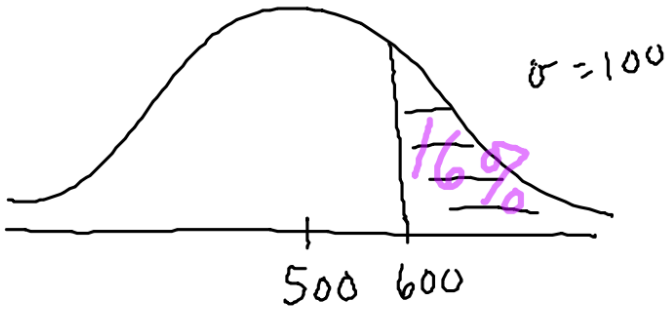


SAT Math Scores $\rightarrow N(500, 100)$;

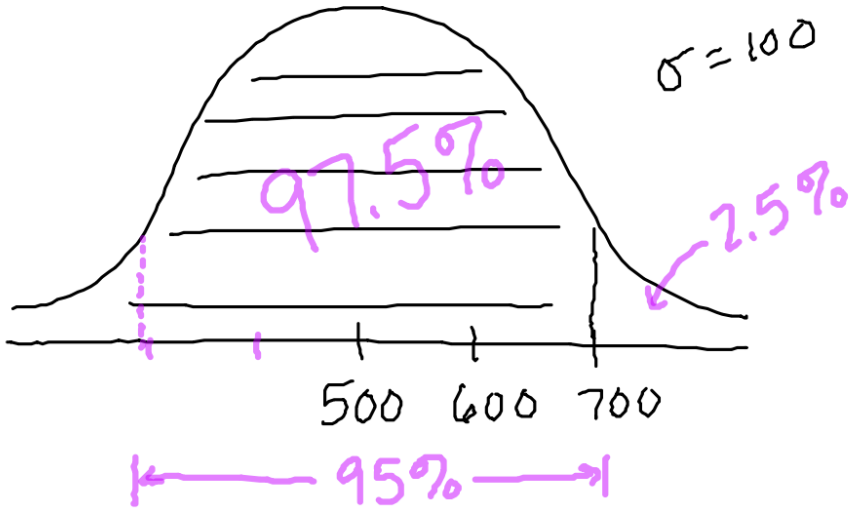
Let $X$ = random SAT Math Score:

$P(400 < X < 600)$

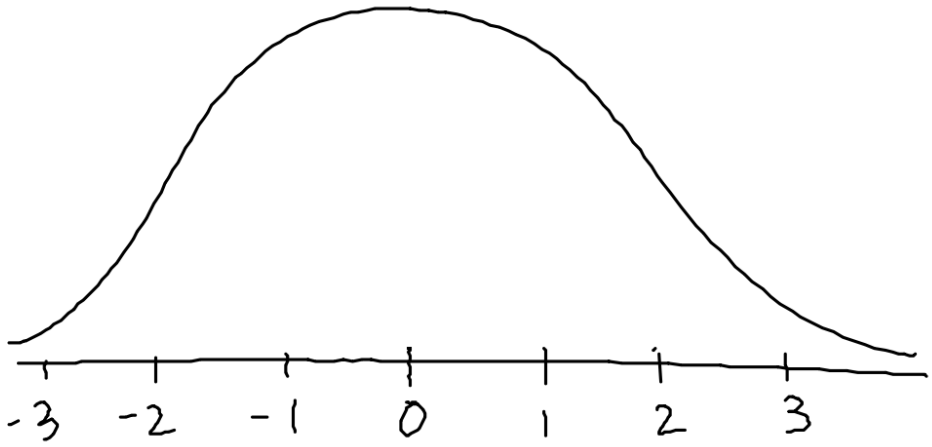

$P(X > 600)$



$P(X < 700)$

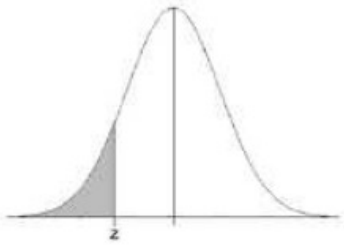


# Standard Normal Curve




Uses Standard Normal Cumulative Probability Table ...

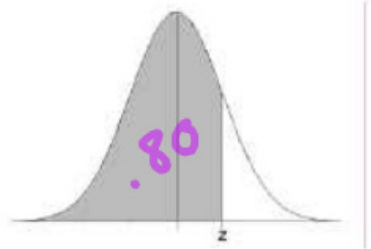
# Standard Normal Cumulative Probability Table




**Cumulative probabilities for NEGATIVE z-values are shown in the following table:**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **-3.4** | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| **-3.3** | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| **-3.2** | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| **-3.1** | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| **-3.0** | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| **-2.9** | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| **-2.8** | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| **-2.7** | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| **-2.6** | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| **-2.5** | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| **-2.4** | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| **-2.3** | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| **-2.2** | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| **-2.1** | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| **-2.0** | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| **-1.9** | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| **-1.8** | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| **-1.7** | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| **-1.6** | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| **-1.5** | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| **-1.4** | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| **-1.3** | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| **-1.2** | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| **-1.1** | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| **-1.0** | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| **-0.9** | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| **-0.8** | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| **-0.7** | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| **-0.6** | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| **-0.5** | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| **-0.4** | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| **-0.3** | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| **-0.2** | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| **-0.1** | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| **0.0** | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |

# Standard Normal Cumulative Probability Table

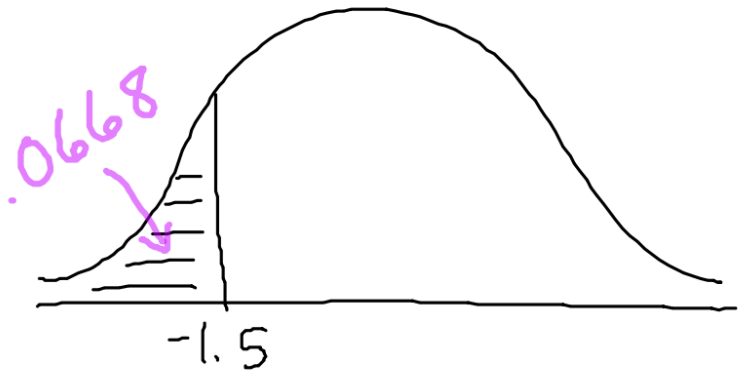



**Cumulative probabilities for POSITIVE z-values are shown in the following table:**

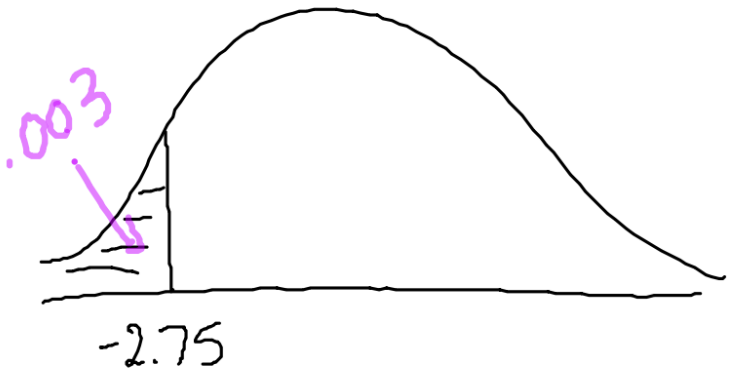
| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

# Find the following probabilities:

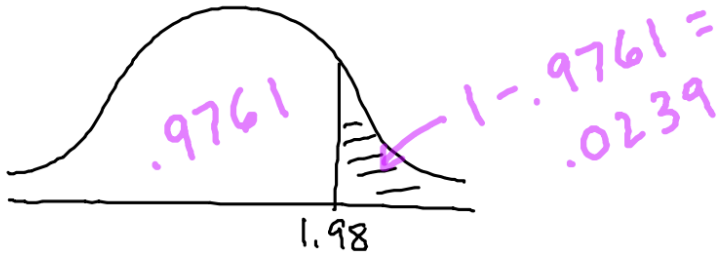
$P(Z < -1.5)$



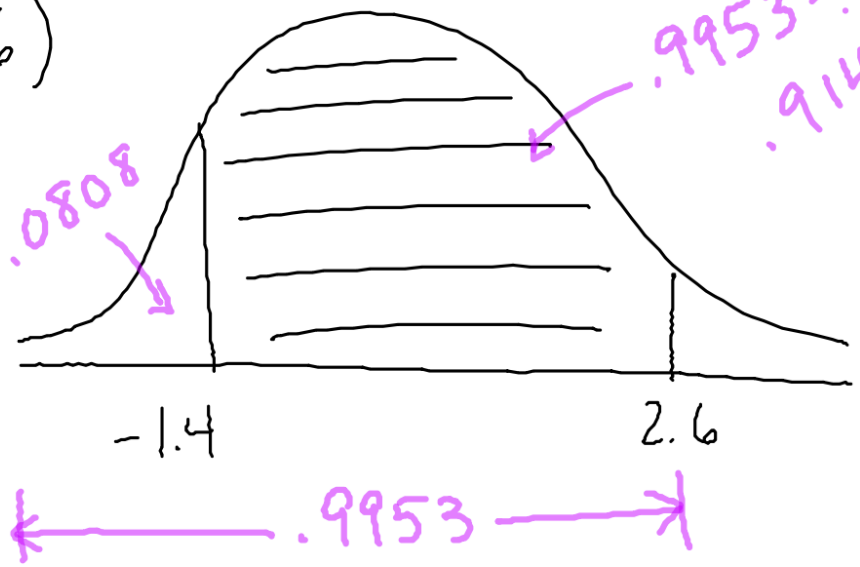

$P(Z < -2.75)$



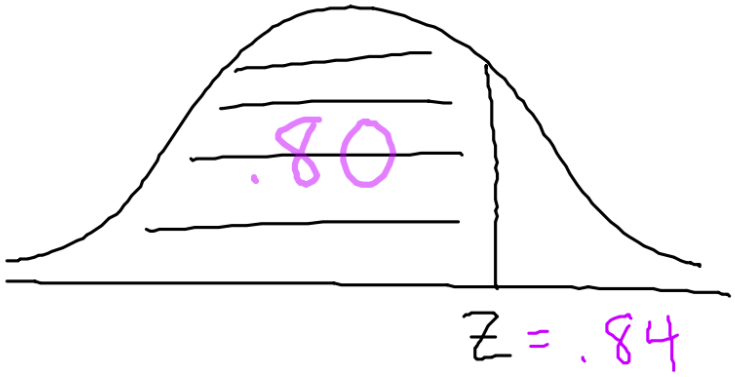
$P(Z > 1.98)$




$P(-1.4 < Z < 2.6)$



IQ Scores → N(100, 15); What IQ puts someone in the 80th Percentile?




$$Z = \frac{X - M}{\sigma}$$

$$.84 = \frac{X - 100}{15}$$

$$X \approx 113$$